

MATH 3322

Supplemental notes on Properties of Normal Subgroups

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A collection of useful odds and ends:

Lemma 0.1 *Normality projects to the bottom: $H \leq K \leq G$, $H \trianglelefteq G$ implies $H \trianglelefteq K$.*

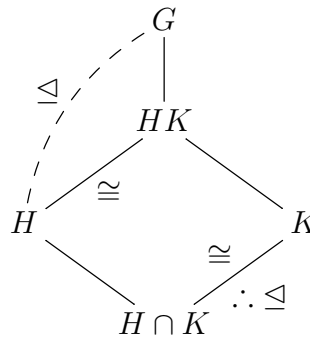
Note that normality is not transitive, and that the other possible implication in this lemma does not hold in general.

Lemma 0.2 *If $H \trianglelefteq G$ and $K \leq G$, then $HK = KH$.*

Lemma 0.3 (*First isomorphism theorem*)
 $H, K \trianglelefteq G$, $H \leq K$, implies $G/K \cong (G/H)/(K/H)$.

Lemma 0.4 (*Parallelogram law/second isomorphism theorem*)
If $H \trianglelefteq HK$ then $H \cap K \trianglelefteq K$, and $HK/K \cong K/H \cap K$.

This is often presented in the context of a group containing H and K , and (combined with Lemma 0.1) in the form of the following diagram:



Lemma 0.5 (*Ames, 4.9, Lemma 7*)

