MATH 2170-19W Problem Set 7

March 20, 2019

Due: in class, March 27, 2019

[8] **Question 1.** Recall that $\mathbb{Z}_7$ denotes the set $\{0, 1, 2, 3, 4, 5, 6\}$ together with the operations of addition and multiplication modulo 7. Recall that every non-zero element of $\mathbb{Z}_7$ has a multiplicative inverse modulo 7:

$$1 \cdot 1 \equiv 1 \pmod 7, \quad 2 \cdot 4 \equiv 1 \pmod 7, \quad 3 \cdot 5 \equiv 1 \quad (\text{mod } 7,) \quad 6 \cdot 6 \equiv 1 \pmod 7$$

Consider the following system of congruences:

$$\begin{cases} 3w + 5x + 5y + 2z & \equiv & 1 \pmod 7 \\ 2w + x + 3y + 5z & \equiv & 4 \pmod 7 \end{cases}$$

Using only the method of Gaussian elimination with back substitution, or the method of Gauss-Jordan elimination, from first year Linear Algebra, [row reduction in matrix form, *no* column operations] and *only* arithmetic in $\mathbb{Z}_7$, find all solutions to this system. Give your solution in vector form.

Question 2.

[2] (a) Find all Pythagorean triples where one of x, y, and z is equal to 17.

[2] (b) Find all primitive Pythagorean triples where $y = 30$, if any.

[4] **Question 3.** Prove that if $\langle x, y, z \rangle$ is a Pythagorean triple, then one of x, y, z is divisible by 3, and one of x, y, z is divisible by 5.

[4] **Question 4.** $g = 2$ is a primitive root modulo 19.

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
g^t	2	4	8	16	13	7	14	9	18	17	15	11	3	6	12	5	10	1

Use this information to calculate the least residue modulo 19 of

(a) $4 \cdot 5 \cdot 7 \cdot 11 \cdot 15 \cdot 17$ (b) 12^{100}

[20] TOTAL