# MATH 2170-19W Problem Set 5 

February 26, 2019
Due: in class, March 06, 2019
[2] Question 1. Find all solutions to

$$
\left\{\begin{array}{llll}
x & \equiv & 1 & (\bmod 3) \\
x & \equiv & 7 & (\bmod 12) \\
x & \equiv 43 & (\bmod 60)
\end{array}\right.
$$

Hint: This question is worth only 2 marks.
[5] Question 2. Find all solutions to $x^{3}+x^{2}+3 x+1 \equiv 0(\bmod 21)$.
[Arithmetic help: $1^{3} \equiv 2^{3} \equiv 4^{3} \equiv 1 \quad(\bmod 7)$ and $\left.3^{3} \equiv 5^{3} \equiv 5^{3} \equiv-1 \quad(\bmod 7).\right]$

## Question 3.

[2] (a) Calculate $\phi(210,000)$.
[3] (b) Find the prime power factorization of $\phi(12!)$.
[3] Question 4. Prove that if $n$ has $k$ distinct odd prime factors, then $2^{k} \mid \phi(n)$.
[5] Question 5. Suppose that $b \equiv a^{53}(\bmod 91)$ and that $(a, 91)=1$. Find a positive number $\bar{k}$ such that $b^{\bar{k}} \equiv a \quad(\bmod 91)$. If $b=67$, what is $a$ ?.

