MATH 2170-19W Problem Set 5

February 26, 2019 Due: in class, March 06, 2019

[2] **Question 1.** Find all solutions to

$$\begin{cases} x \equiv 1 \pmod{3} \\ x \equiv 7 \pmod{12} \\ x \equiv 43 \pmod{60} \end{cases}$$

Hint: This question is worth only 2 marks.

[5] **Question 2.** Find all solutions to $x^3 + x^2 + 3x + 1 \equiv 0 \pmod{21}$. [Arithmetic help: $1^3 \equiv 2^3 \equiv 4^3 \equiv 1 \pmod{7}$ and $3^3 \equiv 5^3 \equiv 5^3 \equiv -1 \pmod{7}$.]

Question 3.

[2] (a) Calculate $\phi(210,000)$.

- [3] (b) Find the prime power factorization of $\phi(12!)$.
- [3] **Question 4.** Prove that if n has k distinct odd prime factors, then $2^k | \phi(n)$.
- [5] **Question 5.** Suppose that $b \equiv a^{53} \pmod{91}$ and that (a, 91) = 1. Find a positive number \overline{k} such that $b^{\overline{k}} \equiv a \pmod{91}$. If b = 67, what is a?.

[20] TOTAL