MATH 2170-19W Problem Set 4

February 14, 2019

Due: in class, February 27, 2019

Question 1.

[2] (a) Let $m > 1$ be an odd natural number. Prove that

$$1 \cdot 3 \cdot 5 \cdot \ldots \cdot (m-2) \equiv (-1)^{\frac{m-1}{2}} \cdot 2 \cdot 4 \cdot 6 \cdot \ldots \cdot (m-1) \pmod{m}$$

[Hint: $1 \equiv -(m-1) \pmod{m}$, $3 \equiv -(m-3) \pmod{m}$, $\ldots$, $m-2 \equiv -2 \pmod{m}$]

[4] (b) If p is an odd prime, prove that

$$1^2 \cdot 3^2 \cdot 5^2 \cdot \ldots \cdot (p-2)^2 \equiv (-1)^{\frac{p+1}{2}} \equiv 2^2 \cdot 4^2 \cdot 6^2 \cdot \ldots \cdot (p-1)^2 \pmod{p}$$

[Hint: Use Part (a), and rearrange the Wilson's Theorem formula in two different ways.]

[2] **Question 2.** Take note that $17 = 1^2 + 4^2 = 4^2 + 1^2$ and $13 = 2^2 + 3^2$
Write $221 = 13 \cdot 17$ as a sum of two squares in two different ways.

[4] **Question 3.** Write each of the following as a sum of two squares.

(a) 1960 (b) $121,000$

[4] **Question 4.** Solve $55x \equiv 91 \pmod{108}$ by solving a pair of congruences, one modulo 4, the other modulo 27.

[4] **Question 5.** Find the smallest positive integer solution to

$$\begin{aligned} x &\equiv 3 \pmod{14} \\ x &\equiv 4 \pmod{15} \\ x &\equiv 5 \pmod{11} \end{aligned}$$

[20] TOTAL