MATH 2170-19W Problem Set 1

January 15, 2019

Due: in class, January 23, 2019

- [3] **Question 1.** Prove: If $a \mid c$ and $b \mid d$, then $ab \mid cd$.
- [8] **Question 2.** An integer n is *even* if $2 \mid n$, and *odd* otherwise. Prove:
- [2] (a) n is even iff n = 2m for some integer m, and n is odd iff n = 2k + 1 for some integer k.
- [1] (b) n is odd iff n + 1 is even.
- [2] (c) n(n+1) is even.
- [3] (d) If n is odd, then $n^2 1$ is divisible by 8.

[6] Question 3.

- [2] (a) Prove that the remainder upon dividing ax + b by a is the same as the remainder upon dividing b by a.
- [3] (b) If we divide an integer n by 5, the possible remainders are 0, 1, 2, 3, 4. What are the possible remainders when n^2 is divided by 5?

[Hint: Set $n = 5k + r, 0 \le r < 5$.]

- [1] (c) Can an integer m ending in 3 be the square of an integer? (explain).
- [3] **Question 4.** Here are diagrams of all the positive divisors of n = 10, n = 20, and n = 42 respectively, drawn as in class. The divisors are arranged according to the divisibility relations: there is a line between two integers if the lower number divides the upper number, and no other divisor fits in between. Furthermore, in a horizontal row, we list the divisors in increasing order.



Draw a similar diagram showing the relations between all the positive divisors of 100. $(100=2^25^2)$ TOTAL

[20]