MATH 1510 TUTORIAL #9

1. (a) $f'(x) = 15x^4 - 15x^2$

$0 = 15x^2(x^2-1) = 15x^2(x-1)(x+1)$ ∴ crit. #'s are: $0, \pm 1$

$f(0) = -1$ $\quad f(-1) = -3+5-1 = 1$ $\quad f(2) = 96-40-1 = 55$

$f(1) = 3-5-1 = -3$ $\quad f(-2) = -96+40-1 = -56$ ∴ ABS. MAX $= 55$

ABS. MIN $= -56$

(b) $f'(x) = 1 + 2\sin x$

$0 = 1 + 2\sin x$

$-\frac{1}{2} = \sin x$

$x = -\frac{5\pi}{6}, -\frac{\pi}{6}$

∴ crit #'s are: $-\frac{5\pi}{6}, -\frac{\pi}{6}$

$f\left(-\frac{5\pi}{6}\right) = -\frac{5\pi}{6} - 2\left(-\frac{\sqrt{3}}{2}\right) = -\frac{5\pi}{6} + \sqrt{3} < 0$

$f\left(-\frac{\pi}{6}\right) = -\frac{\pi}{6} - \left(\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{6} - \frac{\sqrt{3}}{2} < 0$

$f(-\pi) = -\pi + 2 < 0$

$f(\pi) = \pi - 2 > 0$

∴ MAX $= \pi - 2$ $\quad$ MIN $= -\frac{\pi}{6} - \frac{\sqrt{3}}{2}$

(c) $f'(x) = \dfrac{x^2+1-2x(x)}{(x^2+1)^2} = \dfrac{1-x^2}{(x^2+1)^2}$

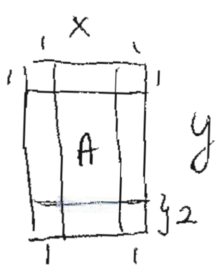
$0 = (1-x)(1+x)$ ∴ critical #'s are: ± 1

$f(1) = \frac{1}{2}$ $\quad f(-1) = -\frac{1}{2}$

$\lim_{x\to\infty} f(x) = 0$ $\quad \lim_{x\to-\infty} f(x) = 0$

∴ MAX $= \frac{1}{2}$ $\quad$ MIN $= -\frac{1}{2}$

2. (a)



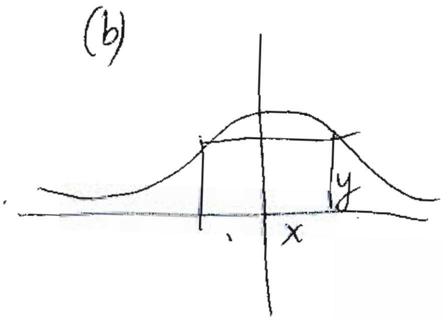
$180 = xy$

$\frac{180}{x} = y$

Printed Area $(A) = (y-3)(x-2)$

$A = \left(\frac{180}{x} - 3\right)(x-2)$ $\quad$ Dom$(0,\infty)$

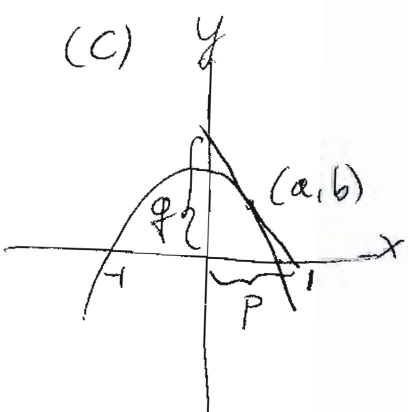
(b)



$A = 2x(y)$ $\quad$ But $y = \frac{1}{x^2+1}$

∴ $A = 2x\left(\frac{1}{x^2+1}\right)$ $\quad$ Dom: $(-\infty, \infty)$

(c)



Area $= \frac{1}{2}pq$.

$y' = -2x$

at (a, b)

$m = -2a$

Equation is:

$y - b = -2a(x-a)$

But at (a,b) $\quad b = 1 - a^2$

∴ $y - (1-a^2) = -2a(x-a)$

∴ $A = \frac{1}{2}\left(\frac{a^2+1}{2a}\right)(a^2-1)$ $\quad$ Domain: $(0, \infty)$

p = x-intercept

$-(1-a^2) = -2ap + 2a^2$

$\frac{-a^2-1}{-2a} = p$

∴ $p = \frac{a^2+1}{2a}$

q = y-intercept

$y - (1-a^2) = 2a^2$

$y = a^2 - 1 = q$.