MATH 1510 Tutorial Worksheet 8
Nov 6/Nov 8, 2007

There are four basic steps to any curve sketching question:

1. What can be learned directly from $f(x)$? What is the domain, what is the range (*range* may be a very difficult question)? Are there points of discontinuity? Is there any symmetry? What are the intercepts? Are there any asymptotes?

2. What can be learned from $f'(x)$? What are the critical points? What are the intervals on which f is increasing? decreasing? Find and classify the local extreme points.

3. What can be learned from $f''(x)$? At what points is $f''(x)$ equal to 0 or undefined? On which intervals is f concave up? concave down? Find any inflection points.

4. Using the information from steps (1)–(3), sketch the graph of $y = f(x)$.

Question 1. Given the following information about $g(x)$, sketch the graph of $y = g(x)$:

g has no symmetry and is not defined at $x = 0$. $g(-1) = -1$, $g(1) = g(3) = 0$, and $g(2) = -1$. $\lim_{x\to-\infty} g(x) = 0$, $\lim_{x\to\infty} g(x) = 1$, $\lim_{x\to0^-} g(x) = -\infty$, and $\lim_{x\to0^+} g(x) = \infty$.

$g'(x) = 0$ only when $x = 2$, and $g'(x)$ is negative when $x < 0$ or $0 < x < 2$. Otherwise $g'(x)$ is positive.

$g''(x) = 0$ only when $x = 3$, and $g''(x)$ is negative when $x < 0$ or when $x > 3$. Otherwise, $g''(x)$ is positive.

Question 2. Sketch the graph of:

(a) $f(x) = \dfrac{x}{x^2 - 1}$

(b) $k(x) = (x^2 - 3x + 2)\mathbf{e}^x$.

You may use the following fact: If $p(x)$ is any polynomial, then $\lim_{x\to-\infty} p(x)\mathbf{e}^x = 0$.

NOTE! Be very careful that you check and double-check your calculations of the first and second derivative! You have plenty of time before the tutorial to prepare your work, and an error at this stage will make the rest of the work pointless.