CALCULUS 1510 TUTORIAL #5

(1.) (a) $\frac{dy}{dx} = \cos(\pi x^2)(2\pi x) - \sin(\sqrt{x})\left(\frac{1}{2\sqrt{x}}\right)$

(b) $\frac{dy}{dx} = \cos(2\cos x)(-2\sin x) + \sec^2(x\sin x)[\sin x + x\cos x]$

(c) $\frac{dy}{dx} = \frac{\cos\sqrt{x}\left(\frac{1}{2\sqrt{x}}\right)[1+\tan\sqrt{x}] - \sin\sqrt{x}\sec^2\sqrt{x}\left(\frac{1}{2\sqrt{x}}\right)}{(1+\tan\sqrt{x})^2}$

(d) $\frac{dy}{dx} = (3x^2 + \sec x\tan x)[\csc(\cot x)] + [x^3 + \sec x](-\csc(\cot x)\cot(\cot x)](-\csc^2$

(e) $3x^2y + x^3\frac{dy}{dx} + y^5 + x(5y^4)\frac{dy}{dx} = 0$

$\frac{dy}{dx}(x^3+5xy^4) = -3x^2y - y^5$

$\frac{dy}{dx} = \frac{-3x^2y - y^5}{x^3+5xy^4}$

(f) $\frac{dy}{dx}\sin x + y\cos x = 3x^2 - \sin y\left(\frac{dy}{dx}\right)$

$\frac{dy}{dx}(\sin x + \sin y) = 3x^2 - y\cos x$

$\frac{dy}{dx} = \frac{3x^2 - y\cos x}{\sin x + \sin y}$

(2.) $y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 2$

$\frac{dy}{dx}(x+2y) = 2-y$

$\frac{dy}{dx} = \frac{2-y}{x+2y}$

$\therefore \frac{d^2y}{dx^2} = \frac{-\frac{dy}{dx} - (2-y)\left(1+2\frac{dy}{dx}\right)}{(x+2y)^2}$

$= \frac{\frac{y-2}{x+2y} + (y-2)\left(1+\frac{2(2-y)}{x+2y}\right)}{(x+2y)^2}$

(3.) $2x + y + x\frac{dy}{dx} + 6y^2\frac{dy}{dx} = 0$

at $(-2,1)$ $\quad 2(-2) + 1 - 2m + 6m = 0$

$4m = 3$

$m = 3/4$

$\therefore$ Equation of Tangent line is:

$y - 1 = \frac{3}{4}(x+2)$

(4.) $\frac{dy}{dx} = 2\cos x(-\sin x)$ when $x = \frac{2\pi}{3}$

$m = 2\cos\left(\frac{2\pi}{3}\right)\left(-\sin\frac{2\pi}{3}\right)$

$= -2\left(-\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2}$

and $y = \left(\cos\frac{2\pi}{3}\right)^2 = \left(-\frac{1}{2}\right)^2 = \frac{1}{4}$

$\therefore$ Equation of tangent line is: $y - \frac{1}{4} = \frac{\sqrt{3}}{2}\left(x - \frac{2\pi}{3}\right)$

(5.) $f'(x) = 2\cos(2x) = 2\cos(2x)$

$f''(x) = -4\sin(2x) = -(2)^2\sin(2x)$

$f'''(x) = -8\cos(2x) = -(2)^3\cos(2x)$

$f^{(4)}(x) = 16\sin(2x) = 2^4\sin(2x)$

$f^{(5)}(x) = 32\cos(2x) = 2^5\cos(2x)$

$f^{(6)}(x) = -64\sin(2x) = -(2)^6\sin 2x$

} a cycle of 4.

$\frac{50}{4} = 12$ Remainder $= 2$

$\therefore f^{(50)}(x) = -(2)^{50}\sin(2x)$