CALCULUS 1510 TUTORIAL #3 SOLUTIONS

(1.) $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} k(x+3) = 6k$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (5k-3) = 5k-3$

For $\lim_{x\to 3} f(x)$ to exist $\lim_{x\to 3^-} f(x)$ must equal $\lim_{x\to 3^-} f(x)$

$\therefore\ 6k = 5k-3$

$k = -3$

When $k=-3$ $\lim_{x\to 3} f(x) = -18 = f(3)$. $\therefore f$ is continuous at $x=3$.

(2.) $\lim_{x\to 2^+} g(x) = \lim_{x\to 2^+} (2x^3-5) = 2(8)-5 = 11$

$\lim_{x\to 2^-} g(x) = \lim_{x\to 2^-} (5x+1) = 10+1 = 11$

Since $\lim_{x\to 2^+} g(x) = \lim_{x\to 2^-} g(x)$ it follows that $\lim_{x\to 2} g(x)$ exists.

Furthermore, $\lim_{x\to 2} g(x) = 11 = g(2)$. Hence g is continuous at $x=2$

(3.) (a) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$$= \lim_{h\to 0} \frac{\sqrt{2(x+h)+3} - \sqrt{2x+3}}{h} \cdot \frac{\sqrt{2(x+h)+3} + \sqrt{2x+3}}{\sqrt{2(x+h)+3} + \sqrt{2x+3}}$$

$$= \lim_{h\to 0} \frac{2(x+h)+3-(2x+3)}{h\left[\sqrt{2x+2h+3} + \sqrt{2x+3}\right]} = \lim_{h\to 0} \frac{2h}{h\left[\sqrt{2x+2h+3} + \sqrt{2x+3}\right]}$$

$$= \lim_{h\to 0} \frac{2}{\sqrt{2x+2h+3} + \sqrt{2x+3}} = \frac{2}{2\sqrt{2x+3}} = \frac{1}{\sqrt{2x+3}}$$

(b) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{-2}{2(x+h)+1} - \left(\frac{-2}{2x+1}\right)}{h} \cdot \frac{(2x+2h+1)(2x+1)}{(2x+2h+1)(2x+1)}$

$$= \lim_{h\to 0} \frac{-2(2x+1)+2(2x+2h+1)}{h(2x+2h+1)(2x+1)} = \lim_{h\to 0} \frac{4h}{h(2x+2h+1)(2x+1)}$$

$$= \lim_{h\to 0} \frac{4}{(2x+2h+1)(2x+1)} = \frac{4}{(2x+1)(2x+1)} = \frac{4}{(2x+1)^2}$$