

MATH 1510 Tutorial Worksheet 12 SOLUTIONS

Evaluate the following integrals.

(a) $\int_{-1}^0 x(x^2-1)^4 dx$

(a) $\frac{(x^2-1)^5}{5(2)} \Big|_{-1}^0 = -\frac{1}{10} - 0 = -\frac{1}{10}$

(b) $\int_0^4 2x\sqrt{4-x} dx$

(b) Let $u = \sqrt{4-x}$
 $u^2 = 4-x$
 $2u du = -dx$

x	u
0	2
4	0

$\int_0^4 2x\sqrt{4-x} dx = 2 \int_2^0 (4-u^2)u(-2u du)$

$= 2 \int_2^0 (-8u^2 + 2u^4) du$

$= 2 \left(-\frac{8}{3}u^3 + \frac{2}{5}u^5 \right) \Big|_2^0 = 0 - \left(-\frac{64}{3} + \frac{64}{5} \right) 2$

(c) $\int_0^{\frac{\pi}{2}} \sin x \sqrt{\cos x} dx$

(d) $\int_0^{\frac{1}{2}} \frac{x}{\sqrt{1-x^2}} dx$

(e) $\int_{-1}^2 (x+1)(2-x)^4 dx$

(f) $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\sec^2 x}{\tan x} dx$

(c) $\frac{-2(\cos x)^{\frac{3}{2}}}{\frac{3}{2}} \Big|_0^{\frac{\pi}{2}} = 0 + \frac{2}{3}$

(d) $-(1-x^2)^{\frac{1}{2}} \Big|_0^{\frac{1}{2}} = -\sqrt{\frac{3}{4}} + \sqrt{1} = 1 - \frac{\sqrt{3}}{2}$

(e) Let $u = 2-x$
 $du = -dx$
 $\therefore x = 2-u$

x	u
-1	3
2	0

$\int_{-1}^2 (x+1)(2-x)^4 dx = \int_3^0 (3-u)u^4(-du) = -\int_3^0 (3u^4 - u^5) du$
 $= -\left(\frac{3}{5}u^5 - \frac{u^6}{6} \right) \Big|_3^0 = 0 - \left(-\frac{729}{5} + \frac{729}{6} \right)$

(f) $\ln|\tan x| \Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}} = \ln 1 - \ln\left(\frac{1}{\sqrt{3}}\right) = 0 - (\ln 1 - \ln \sqrt{3}) = \ln \sqrt{3} \text{ or } \frac{1}{2} \ln 3.$

Note: Substitution could be used in (a), (c), (d) and (f).

(a) Let $u = x^2-1$
 $du = 2x dx$

x	u
-1	0
0	-1

$\int_0^{-1} \frac{u^4}{2} du = \frac{u^5}{10} \Big|_0^{-1} = -\frac{1}{10}$

(c) Let $u = \sqrt{\cos x}$
 $u^2 = \cos x$
 $2u du = -\sin x dx$

x	u
0	1
$\frac{\pi}{2}$	0

$\int_1^0 u(-2u du) = -\int_1^0 2u^2 du = -\frac{2u^3}{3} \Big|_1^0 = 0 - \left(-\frac{2}{3} \right) = \frac{2}{3}$

(d) Let $u = \sqrt{1-x^2}$
 $u^2 = 1-x^2$
 $2u du = -2x dx$

x	u
0	1
$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$

$\int_1^{\frac{\sqrt{3}}{2}} \frac{-u du}{u} = \int_1^{\frac{\sqrt{3}}{2}} -du = -u \Big|_1^{\frac{\sqrt{3}}{2}} = -\frac{\sqrt{3}}{2} + 1$

(f) Let $u = \tan x$
 $du = \sec^2 x dx$

x	u
$\frac{\pi}{6}$	1
$\frac{\pi}{4}$	$\frac{1}{\sqrt{3}}$

$\int_{\frac{1}{\sqrt{3}}}^1 \frac{du}{u} = \ln|u| \Big|_{\frac{1}{\sqrt{3}}}^1 = \ln 1 - \ln\left(\frac{1}{\sqrt{3}}\right) = \ln 1 - \ln 1 + \ln \sqrt{3} = \ln \sqrt{3} \text{ or } \frac{1}{2} \ln 3$