MATH 1510 Tutorials November 27, 29
Substitutions in Indefinite Integrals
Basic Patterns

Question 1. (Linear substitutions)

$$\int (5x+7)^{12}\,dx = \frac{1}{5}\frac{(5x+7)^{13}}{13} + C$$

$$\int (\sqrt{2}t - \sqrt{3})^{5/3}\,dt = \frac{1}{\sqrt{2}}\frac{(\sqrt{2}t-\sqrt{3})^{8/3}}{(8/3)} + C$$

$$\int \sqrt{\pi x + 2}\,dx = \frac{1}{\pi}\frac{2}{3}(\pi x + 2)^{(3/2)} + C$$

$$\int \sin\left(\frac{\pi}{3}\theta + \frac{\pi}{4}\right)\,d\theta = -\frac{3}{\pi}\cos\left(\frac{\pi}{3}\theta + \frac{\pi}{4}\right) + C$$

$$\int \frac{1}{2x-5}\,dx = \frac{1}{2}\ln(2x-5) + C$$

$$\int \mathbf{e}^{3t-2}\,dt = \frac{1}{3}\mathbf{e}^{3t-2} + C$$

Question 2. (Logarithms) [Substitute $u =$ *denominator*]

$$\int \frac{x}{x^2+1}\,dx = \frac{1}{2}\ln(x^2+1) + C$$

$$\int \frac{x^2+1}{x^3+3x+1}\,dx = \frac{1}{3}\ln(\left| x^3+3x+1 \right|) + C$$

$$\int \frac{u^{1/3}}{u^{4/3}+5}\,du = \frac{3}{4}\ln(u^{4/3}+5) + C$$

$$\int \frac{\mathbf{e}^{2t}}{\mathbf{e}^{2t}+4}\,dt = \frac{1}{2}\ln(\mathbf{e}^{2t}+4) + C$$

$$\int \frac{\sin(\theta)}{\cos(\theta)}\,d\theta = -\ln|\cos(\theta)| + C = \ln\left|\frac{1}{\cos(\theta)}\right| + C = \ln|\sec(\theta)| + C$$

$$\int \frac{1}{x\ln x}\,dx = \ln|\ln(x)| + C \qquad (u = \ln(x))$$

Question 3. (Square roots) ($u = \sqrt{x}$, or $u^2 = x$, or some close variant)

$$\int \frac{\mathbf{e}^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\mathbf{e}^{\sqrt{x}} + C$$

$$\int \frac{\cos(\sqrt{t})}{\sqrt{t}}\,dt = 2\sin(\sqrt{t}) + C$$

$$\int \frac{(1+\sqrt{x})^{5/3}}{\sqrt{x}}\,dx = 2\cdot\frac{3}{8}(1+\sqrt{x})^{(8/3)} + C$$

$$\int \frac{\mathbf{e}^{\sqrt{x+5}}}{\sqrt{x+5}}\,dx = 2\mathbf{e}^{\sqrt{x+5}} + C$$

$$\int \frac{\sin(\sqrt{\theta}+\pi)}{\sqrt{\theta}}\,d\theta = -2\cos(\sqrt{\theta}+\pi) + C$$

$$\int \frac{1}{\sqrt{x}\sqrt{1+\sqrt{x}}}\,dx = 4\sqrt{1+\sqrt{x}}$$

Question 4. **(Square roots)** ($u =$ *thing under the square root*, or $u^2 =$ *thing under the square root*)

$$\int \frac{f'(x)}{\sqrt{f(x)}}\,dx \qquad (u^2 = f(x),\ 2u\,du = f'(x)\,dx)$$

$$= \int \frac{2u\,du}{u}$$

$$= \int 2\,du$$

$$\int \frac{\mathbf{e}^t}{\sqrt{\mathbf{e}^t+2}}\,dt = 2\sqrt{\mathbf{e}^t+2}+C$$

$$\int \frac{\cos(\theta)}{\sqrt{1+\sin(\theta)}}\,d\theta == 2\sqrt{1+\sin(\theta)}+C$$

$$\int \frac{x}{\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2}+C$$

$$\int \frac{1}{x\sqrt{\ln(x)}}\,dx = 2\sqrt{\ln(x)}+C$$

$$\int \frac{2ax+b}{\sqrt{ax^2+bx+c}}\,dx = 2\sqrt{ax^2+bx+c}+C$$

$$\int \frac{\sin(\theta)\cos(\theta)}{\sqrt{1+\sin^2(\theta)}}\,d\theta = \sqrt{1+\sin^2(\theta)}+C$$

Question 5. **Miscellaneous patterns**

$$\int t\mathbf{e}^{t^2+1}\,dt = \frac{1}{2}\mathbf{e}^{t^2+1}+C \qquad (u = t^2+1)$$

$$\int x(x^2+1)^{10}\,dx = \frac{1}{2}\frac{1}{11}(x^2+1)^{11}+C \qquad (u = x^2+1)$$

$$\int \theta\cos(\theta^2+\pi)\,d\theta = \frac{1}{2}\sin(\theta^2+\pi)+C \qquad (u = \theta^2+\pi)$$

$$\int y\sqrt{y^2+1}\,dy = \frac{1}{3}(y^2+1)+C \qquad (u = y^2+1)$$

$$\int \cos(t)\mathbf{e}^{\sin(t)}\,dt = \mathbf{e}^{\sin(t)}+C \qquad (u = \sin(t))$$

$$\int \sin(x)\cos^{10}(x)\,dx = -\frac{1}{11}\cos(x)^{11}+C \qquad (u = \cos(t))$$

$$\int \cos(\theta)\cos(\sin(\theta))\,d\theta = \sin(\sin(\theta))+C \qquad (u = \sin(\theta)$$

$$\int \sin(y)\sqrt{\cos(y)+1}\,dy = -\frac{2}{3}(\cos(y)+1)^{(3/2)}+C \qquad (u = \cos(y)+1)$$

$$\int \mathbf{e}^t\mathbf{e}^{\mathbf{e}^t+1}\,dt = \mathbf{e}^{\mathbf{e}^t+1}+C \qquad (u = \mathbf{e}^t+1)$$

$$\int \mathbf{e}^x(\mathbf{e}^x+1)^{10}\,dx = \frac{1}{11}(\mathbf{e}^x+1)^{11}+C \qquad (u = \mathbf{e}^x+1)$$

$$\int \mathbf{e}^\theta\cos(\mathbf{e}^\theta+1)\,d\theta = \sin(\mathbf{e}^\theta+1)+C \qquad (u = \mathbf{e}^\theta+1)$$

$$\int \mathbf{e}^y\sqrt{\mathbf{e}^y+1}\,dy = \frac{2}{3}(\mathbf{e}^y+1)^{3/2}+C \qquad (u = \mathbf{e}^y+1)$$