MATH 1510 Tutorials November 27, 29
Substitutions in Indefinite Integrals
Basic Patterns

Searching for patterns: Skill with the method of substitution requires the ability to recognize the "derivative obtained by the chain rule" pattern. The first practice step should be to work out many simple chain rule differentiations to fix these patterns in your mind. What follows are five straightforward practice exercises running through some common patterns.

Question 1. **(Linear substitutions)** Pattern: $\dfrac{d}{dx}f(ax+b) = a \cdot f'(ax+b)$, where a and b are constants. Therefore $\int f'(ax+b)\,dx = \frac{1}{a}f(ax+b)$. Learn to recognize the pattern $g(ax+b)$ in an integrand and you may simplify it immediately by the linear substitution $u = ax+b$.

$\int (5x+7)^{12}\,dx \qquad \int (\sqrt{2}t - \sqrt{3})^{5/3}\,dt \qquad \int \sqrt{\pi x + 2}\,dx$

$\int \sin\left(\frac{\pi}{3}\theta + \frac{\pi}{4}\right)\,d\theta \qquad \int \frac{1}{2x-5}\,dx \qquad \int \mathbf{e}^{3t-2}\,dt$

Question 2. **(Logarithms)** Pattern: $\dfrac{d}{dx}\ln(f(x)) = \dfrac{f'(x)}{f(x)}$.

$\int \frac{x}{x^2+1}\,dx \qquad \int \frac{x^2+1}{x^3+3x+1}\,dx \qquad \int \frac{u^{1/3}}{u^{4/3}+5}\,du$

$\int \frac{\mathbf{e}^{2t}}{\mathbf{e}^{2t}+4}\,dt \qquad \int \frac{\sin(\theta)}{\cos(\theta)}\,d\theta \qquad \int \frac{1}{x\ln x}\,dx$

Question 3. **(Square roots)** First Pattern: $\dfrac{d}{dx}f(\sqrt{x}) = \dfrac{f'(\sqrt{x})}{2\sqrt{x}}$.

You can simplify by letting $u = \sqrt{x}$ and writing $u^2 = x$ *before* differentiating.

$\int \frac{\mathbf{e}^{\sqrt{x}}}{\sqrt{x}}\,dx \qquad \int \frac{\cos(\sqrt{t})}{\sqrt{t}}\,dt \qquad \int \frac{(1+\sqrt{x})^{5/3}}{\sqrt{x}}\,dx$

$\int \frac{\mathbf{e}^{\sqrt{x+5}}}{\sqrt{x+5}}\,dx \qquad \int \frac{\sin(\sqrt{\theta}+\pi}{\sqrt{\theta}}\,d\theta \qquad \int \frac{1}{\sqrt{x}\sqrt{1+\sqrt{x}}}\,dx$

Question 4. **(Square roots)** Second Pattern: $\dfrac{d}{dx}\sqrt{f(x)} = \dfrac{f'(x)}{2\sqrt{f(x)}}$.

$\int \frac{\mathbf{e}^t}{\sqrt{\mathbf{e}^t+2}}\,dt \qquad \int \frac{\cos(\theta)}{\sqrt{1+\sin(\theta)}}\,d\theta \qquad \int \frac{x}{\sqrt{1-x^2}}\,dx$

$\int \frac{1}{x\sqrt{\ln(x)}}\,dx \qquad \int \frac{2ax+b}{\sqrt{ax^2+bx+c}}\,dx \qquad \int \frac{\sin(\theta)\cos(\theta)}{\sqrt{1+\sin^2(\theta)}}\,d\theta$

Question 5. **Miscellaneous patterns**

$\int t\mathbf{e}^{t^2+1}\,dt \qquad \int x(x^2+1)^{10}\,dx \qquad \int \theta\cos(\theta^2+\pi)\,d\theta \qquad \int y\sqrt{y^2+1}\,dy$

$\int \cos(t)\mathbf{e}^{\sin(t)}\,dt \qquad \int \sin(x)\cos^{10}(x)\,dx \qquad \int \cos(\theta)\cos(\sin(\theta))\,d\theta \qquad \int \sin(y)\sqrt{\cos(y)+1}\,dy$

$\int \mathbf{e}^t\mathbf{e}^{\mathbf{e}^t+1}\,dt \qquad \int \mathbf{e}^x(\mathbf{e}^x+1)^{10}\,dx \qquad \int \mathbf{e}^\theta\cos(\mathbf{e}^\theta+1)\,d\theta \qquad \int \mathbf{e}^y\sqrt{\mathbf{e}^y+1}\,dy$