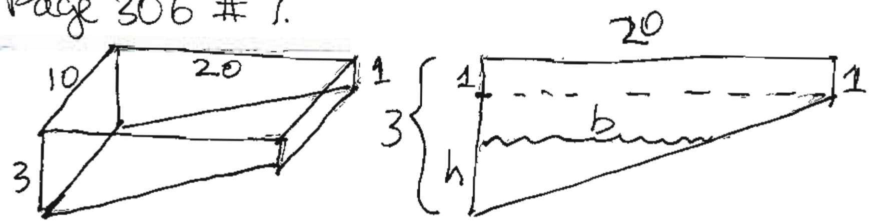
Math 1510 Tutorials Nov 20/22

Page 306 #7.



Let h (m) be the depth of the water. Let b (m) be the length of the exposed surface area of the water.

Let V (m^3) be the volume of water in the pool. h, b, V are functions of time t (m). $V =$ ~~area of~~ triangular cross-section $\times$ (width $= 10m$)

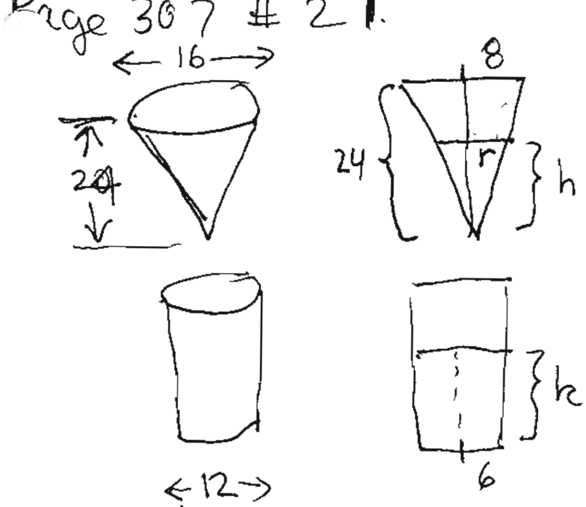
$V = \frac{1}{2} bh \cdot 10$; By similar triangles $\frac{b}{h} = \frac{20}{2}$, $b = 10h$

so $V = \frac{1}{2} 10 \cdot h \cdot h \cdot 10 = 50h^2$. Question When $h = 1m$, $\frac{dh}{dt} = .01$ m/min [Not 1 cm/min!] What is dV/dt?

Solution: $\frac{dV}{dt} = 50 \cdot 2h \frac{dh}{dt} = 100 \cdot 1 \cdot (.01) = 1$.

The water is being pumped in to the pool at $1\ m^3/min$.

Page 307 #21.



All lengths are in cm.

Let h be the depth of the ~~wat~~ solution in the filter; let k be the depth of the solution in the cylinder. Let r be the radius of the surface of the solution in the filter, let V (cm^3) be the volume of the solution in the filter, let W be the volume of the solution in the cylinder.

All these quantities vary with respect to time t (min).

By similar triangles, $\frac{r}{h} = \frac{8}{24}$, so $3r = h$, $r = h/3$

$V = \frac{1}{3}\pi r^2 h$; $W = \pi \cdot 6^2 \cdot k = 36\pi k$; $V = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h = \frac{\pi}{27} h^3$

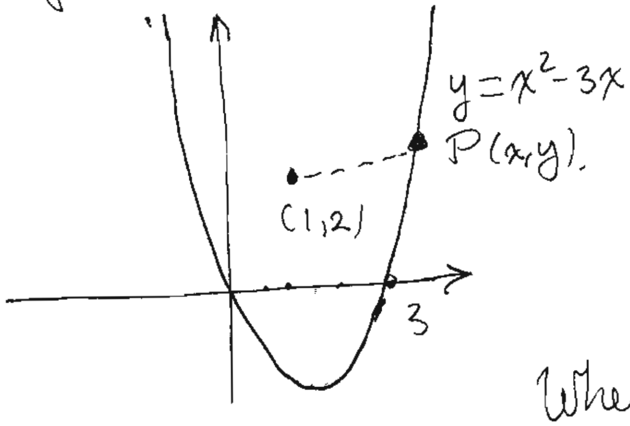
~~Wa~~ Solution is being added to the cylinder at the same rate as it drips out of the filter, so $\frac{dW}{dt} = -\frac{dV}{dt}$.

Problem: when $h = 12$, $\frac{dh}{dt} = -1$. What is $\frac{dk}{dt}$?

$\frac{dW}{dt} = 36\pi \frac{dk}{dt}$, $\frac{dV}{dt} = \frac{\pi}{27} 3h^2 \frac{dh}{dt}$. Therefore $\frac{dV}{dt} = \frac{\pi}{9} \cdot 144(-1)$

and so $\frac{dW}{dt} = -\left(\frac{\pi}{9} \cdot 144 \cdot (-1)\right) = 16\pi = 36\pi \frac{dk}{dt}$, $\frac{dk}{dt} = \frac{16}{36} = \frac{4}{9}$

Page 307 #23. Math 1510 Tutorials Nov 20/22



Let z (m) be the distance between P and $(1,2)$.

So $z^2 = (x-1)^2 + (y-2)^2$ (distance formula).

When $x=4$, $y=4$, and

$z^2 = 3^2 + 2^2 = 13$, $z = \sqrt{13}$

Problem When $x=4$, $y=4$, ~~an~~ $\frac{dx}{dt} = 2$. What is $\frac{dz}{dt}$?

$$z^2 = (x-1)^2 + (y-2)^2$$

$$2z\frac{dz}{dt} = 2(x-1)\frac{dx}{dt} + 2(y-2)\frac{dy}{dt}$$

$$z\frac{dz}{dt} = (x-1)\frac{dx}{dt} + (y-2)\frac{dy}{dt}$$

$y = x^2 - 3x$

$\frac{dy}{dt} = (2x-3)\frac{dx}{dt}$

$x=4$, $y=4$, $z=\sqrt{13}$, $\frac{dx}{dt} = 2$. So $\frac{dy}{dt} = (2\cdot 4 - 3)2 = 10$

and $\sqrt{13}\frac{dz}{dt} = \cancel{3\frac{dx}{dt}}\ 3\cdot 2 + 2\cdot 10 = 26$.

$$\frac{dz}{dt} = \frac{26}{\sqrt{13}} \left(= \frac{2\cdot 13}{\sqrt{13}} = 2\sqrt{13}\right)$$

The distance from P to $(1,2)$ is changing at $2\sqrt{13}$ m/s.