

Let $AB = d$

PROBLEM SET #4 CH 1510

$$\frac{da}{dt} = 30 \text{ m/hr} \quad \frac{db}{dt} = 20 \text{ m/hr} \quad \frac{dd}{dt} = ?$$

$$d^2 = (a+b)^2 + 40^2$$

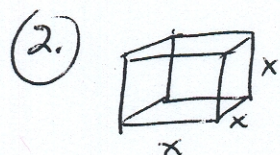
$$2d \frac{dd}{dt} = 2(a+b) \left(\frac{da}{dt} + \frac{db}{dt} \right) + 0$$

Correction
velocities in m/hr.
at 2 PM

$$d^2 = (30+20)^2 + 40^2 = 50^2 + 40^2 = 10^2(5^2 + 4^2) \therefore d = 10\sqrt{41}$$

$$2(10\sqrt{41}) \frac{dd}{dt} = 2(30+20)(30+20) \therefore \frac{dd}{dt} = \frac{(50)^2}{10\sqrt{41}} = \frac{250}{\sqrt{41}}$$

The cars are separating at $\frac{250}{\sqrt{41}}$ m/hr



$$\frac{dA}{dt} = -48 \text{ cm}^2/\text{s}$$

$$\frac{dV}{dt} = ?$$

$$A = 6x^2$$

$$V = x^3$$

$$\frac{dA}{dt} = 12x \frac{dx}{dt}$$

$$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$$

at the moment $x = 12 \text{ cm}$.

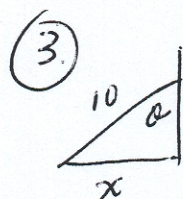
$$-48 = 12(12) \frac{dx}{dt}$$

$$\frac{dV}{dt} = 3(12)^2 \left(\frac{-1}{3} \right)$$

$$-\frac{1}{3} = \frac{dx}{dt}$$

$$\frac{dV}{dt} = -144$$

$\therefore$ the volume is decreasing at $144 \text{ cm}^3/\text{s}$.



$$\frac{dx}{dt} = 2 \text{ m/s}$$

$$\frac{d\theta}{dt} = ?$$

$$\sin \theta = \frac{x}{10}$$

$$\cos \theta \frac{d\theta}{dt} = \frac{1}{10} \frac{dx}{dt}$$

$$\text{When } \theta = \frac{\pi}{4}$$

$$\frac{\sqrt{2}}{2} \frac{d\theta}{dt} = \frac{1}{10} (2)$$

$$\cos \theta = \frac{\sqrt{2}}{2}$$

$$\frac{d\theta}{dt} = \frac{\sqrt{2}}{5}$$

The angle is increasing at $\frac{\sqrt{2}}{5}$ radians/s.

(4) (a) $\int \frac{x^2}{6} - \frac{x}{2} dx = \frac{x^3}{18} - \frac{x^2}{4} + C$

(b) $\frac{2e^{3x}}{3} + \ln|x| - x \cos x + C$

(c) $\frac{(2x+3)^{3/2}}{3} + C$

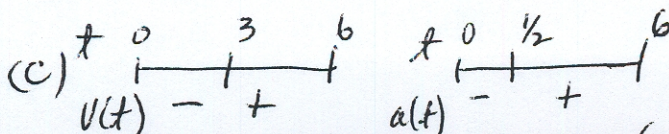
(5) (a) $x(t) = \frac{t^3}{3} - \frac{t^2}{2} - 6t + C$ But $x(0) = 3 \therefore C = 3$

$$\therefore x(t) = \frac{t^3}{3} - \frac{t^2}{2} - 6t + 3$$

$$x(6) = 72 - 18 - 36 + 3 = 21 \text{ m}$$

(b) $x'(t) = v(t) = t^2 - t - 6 = (t-3)(t+2) \therefore$ in the interval $[0, 6]$ the object turns around at $t = 3$ and $x(3) = 9 - \frac{9}{2} - 18 + 3$

$$\text{Distance} = |f(3) - f(0)| + |f(6) - f(3)| = |-10\frac{1}{2} - 3| + |21 - (-10\frac{1}{2})| = 13\frac{1}{2} + 31\frac{1}{2} = 45 \text{ m.}$$



$$a(t) = v'(t) = 2t - 1$$

speeding up on $(0, \frac{1}{2})$ since $v(t) < 0$ & $a(t) < 0$
speeding up on $(3, 6)$ since $v(t) > 0$ & $a(t) > 0$. But slowing down on $(\frac{1}{2}, 3)$ since $v(t) < 0$ & $a(t) > 0$.

MATH 1510 Problem Set 4. Solutions