MATH 1510, PROBLEM SET 2

(1.) (a) $f'(x) = 3(x+\cos(\pi x))^2(1-\sin(\pi x)\pi)$

(b) $f'(x) = 5^{x^2+x}(2x+1)\ln 5 \log_3 x + 5^{x^2+x}\left(\frac{1}{x\ln 10}\right)$

(c) $f'(x) = \frac{(\sec^2 x + e^{-x})(e^{3x}) - (\tan x - e^{-x})(3e^{3x})}{(e^{3x})^2}$

(d) $f'(x) = y' + 3\sin^2(x^2)(\cos x^2)(2x)$ where $y=(x+1)^{\ln x}$

$\ln y = \ln x \ln(x+1)$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x}\ln(x+1) + \frac{\ln x}{x+1}$

$\frac{dy}{dx} = y\left[\frac{\ln(x+1)}{x} + \frac{\ln x}{x+1}\right]$

$\frac{dy}{dx} = (x+1)^{\ln x}\left[\frac{\ln(x+1)}{x} + \frac{\ln x}{x+1}\right]$

$\therefore f'(x) = (x+1)^{\ln x}\left[\frac{\ln(x+1)}{x} + \frac{\ln x}{x+1}\right] + 3\sin^2(x^2)\cos(x^2)(2x)$

(2.) $\lim\limits_{x\to 0} \frac{(x^2-1)\sin^2(\pi x)}{x^2}\cdot\frac{\pi^2}{\pi^2}$

$= \lim\limits_{x\to 0} \pi^2(x^2-1) \lim\limits_{x\to 0} \frac{(\sin \pi x)(\sin \pi x)}{(\pi x)(\pi x)} = \pi^2(-1)(1)(1) = -\pi^2$

(3.) $f'(x) = 3e^{3x}$

$f''(x) = 9e^{3x} = 3^2 e^{3x}$

$f'''(x) = 3^3 e^{3x}$

$\therefore f^{(21)}(x) = 3^{21}e^{3x}$

(4.) $2e^{-x}(-1) + e^y\frac{dy}{dx} = 3e^{x-y}\left(1-\frac{dy}{dx}\right)$

at $(0,0)$

$2(1)(1) + 1m = 3(1)(1-m)$

$2+m = 3-3m$

$4m = 1$

$m = 1/4$

$\therefore$ Equation of tangent line is: $y = \frac{1}{4}x$

(5.) $\frac{dy}{dx}e^{xy} + ye^{xy}\left(1y + x\frac{dy}{dx}\right) = 0$

at $(\ln 2, 1)$

$me^{\ln 2} + 1e^{\ln 2}(1+\ln 2(m)) = 0$

$m(2) + 2(1+m\ln 2) = 0$

$2m + 2 + m\ln 2 = 0$

$m(2+\ln 2) = -2$

$m = \frac{-2}{2+\ln 2}$

$m_\perp = \frac{2+\ln 2}{2}$

$\therefore$ Equation of normal line is:

$y - 1 = \left(\frac{2+\ln 2}{2}\right)(x - \ln 2)$