MATH 1510 Problem Set 3

November 6, 2007

Due: November 19, 20, 2007

This assignment will receive a critical grading of some questions (or parts of questions) **if you hand it in to your instructor in class on the date specified above.** Solutions to all the questions will be posted to the course website after the due date. You do not receive any credit for this assignment and no records are kept. Participation is voluntary but highly recommended!

Question 1. Find the absolute maximum and absolute minimum (if any) of each function on the interval indicated:

- (a) $f(x) = x^4 4x^2 + 2$, on the interval [-1, 3],
- (b) $g(x) = x 2\cos(x)$, on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$,
- (c) $h(x) = (x-1)^2 e^{-x}$ on the interval $[0, \infty)$. (Remember that if p(x) is any polynomial then $\lim_{x \to \infty} p(x) e^{-x} = 0$.)

Question 2. Set up the equation(s) required to solve the following word problems. Draw any diagrams necessary, identifying variables, and clearly state the function to be maximized or minimized, with any constraints on the variables. DO NOT SOLVE THE PROBLEM!

- (a) The illumination of an object by a light source is directly proportional to the strength of the source and inversely proportional to the square of the distance from the source. If two light sources, one three times as strong as the other, are placed 10 m apart, where should an object be placed on the line between the two sources so as to receive the minimum possible illumination?
- (b) A long piece of sheet-metal 0.5 m wide is to be bent in half lengthwise to form a trough whose cross-section is an isosceles triangle. What is the angle at the bottom of the trough that gives the maximum possible cross-sectional area, and therefore the greatest volume for the trough?

Question 3.

- (a) An object moves along the x-axis so that its position at time t, $0 \le t \le 1$ is given by $x(t) = 9t^4 16t^3 + 8t^2$, where position is given in metres and time is given in seconds. Find the following:
 - 1. The time intervals on which the velocity is increasing and on which the velocity is decreasing.
 - 2. The time intervals on which the speed is increasing and on which the speed is decreasing.
 - 3. The maximum and the minimum value of the velocity on the time interval $-\frac{1}{3} \le t \le 1$.
 - 4. The maximum and the minimum value of the speed on the time interval $-\frac{1}{3} \le t \le 1$.
 - 5. The average velocity over the time interval $-\frac{1}{3} \le t \le 1$.

Remark: The numbers have been chosen so that most of the arithmetic works out easily!

- (b) An object moves along the x-axis so that its position at time $t, t \ge 0$, is given by a function $x(t) = at^3 + bt^2 + ct + d$. It starts at the origin with velocity 2 m/s and acceleration -6 m/s^2 , and returns to the origin at time t = 2 s.
 - 1. Find the values of a, b, c, and d.
 - 2. Does the object pass through the origin at any other time?
 - 3. What is the speed of the object at each time that it passes through the origin?
 - 4. What is the acceleration of the object at each time that it passes through the origin?
 - 5. On what interval (if any) is the object moving from right to left?