

INSTRUCTIONS

- I. No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.
- II. This exam has a title page, 4 pages of questions and two blank pages for rough work. Please check that you have all the pages. **DO NOT REMOVE THE SCRAP PAPER**
- III. The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 50 points.
- IV. **Answer all questions on the exam paper** in the space provided beneath the question. Unjustified answers will receive little or no credit. Only techniques taught in this course should be used. **Do not continue on the back of the page.** If you need more space, continue on one of the scrap pages, CLEARLY INDICATING THAT YOUR WORK IS TO BE CONTINUED.
- V. Do not deface the QR - code in the top right corner. Doing so may result in the page not being scanned and therefore not graded.

Question	Points	Score
1	4	
2	2	
3	7	
4	6	
5	9	
6	22	
Total:	50	

- [4] 1. Let $h(x) = \frac{2g(x) + 1}{f(x) - 1}$, Find $h'(5)$ if $f'(5) = g'(5) = 1$ and $f(5) = g(5) = -1$.

Solution:

Using the quotient rule for derivatives,

$$\begin{aligned} h'(x) &= \frac{(2g(x) + 1)'(f(x) - 1) - (2g(x) + 1)(f(x) - 1)'}{(f(x) - 1)^2} \\ &= \frac{2g'(x)(f(x) - 1) - (2g(x) + 1)f'(x)}{(f(x) - 1)^2} \end{aligned}$$

Therefore,

$$\begin{aligned} h'(5) &= \frac{2g'(5)(f(5) - 1) - (2g(5) + 1)f'(5)}{(f(5) - 1)^2} \\ &= \frac{2(-1)((-1) - 1) - (2(-1) + 1)(1)}{((-1) - 1)^2} = -\frac{3}{4} \end{aligned}$$

- [2] 2. Given that, for a function $f(x)$, $f''(x)$ is continuous near $x = c$, $f'(c) = 0$, and $f''(c) > 0$, what can we conclude about $f(c)$ in terms of relative (local) extrema? Explain your answer.

Solution: Since $f'(c) = 0$, f is defined at $x = c$ and has a critical number at $x = c$. Since $f''(c) > 0$, according to the second derivative test, f has a relative (local) minimum at $x = c$.

- [7] 3. Find the absolute extrema of $f(x) = 4x^3 - x^4 + 10$ on the interval $[-1, 4]$. Justify your answer.

Solution: Since f is a polynomial function, it is continuous on the closed interval $[-1, 4]$.

$$f'(x) = 12x^2 - 4x^3$$

which is defined everywhere.

To find critical numbers, we have to solve $f'(x) = 0$.

$$f'(x) = 0 \Rightarrow 12x^2 - 4x^3 = 0 \Rightarrow 4x^2(3 - x) = 0 \Rightarrow x = 0 \text{ or } x = 3.$$

Hence, with the endpoints, we test $x = 0$ and $x = 3$, since they are both in the interval.

$$f(-1) = 5$$

$$f(0) = 10$$

$$f(3) = 37$$

$$f(4) = 10.$$

Then, the absolute maximum is 37 and the absolute minimum is 5.

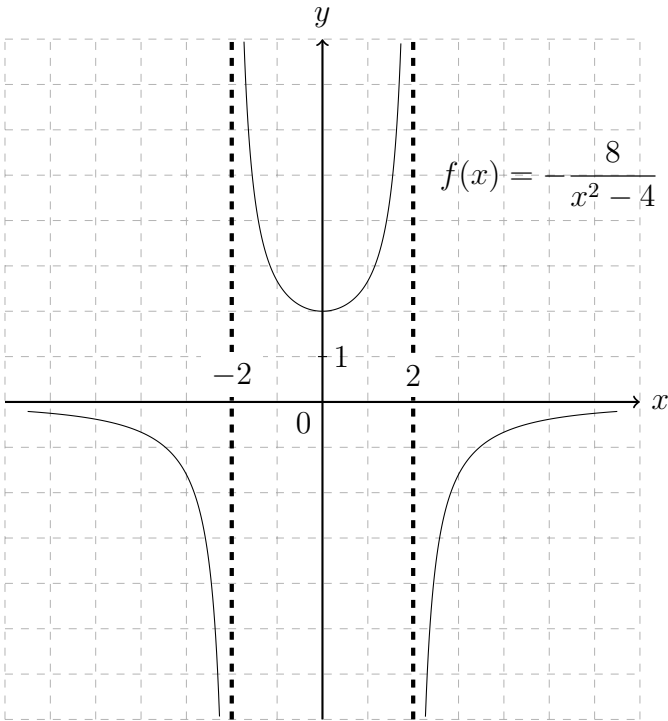
- [6] 4. Suppose that $f(x)$ is differentiable on an interval I and $f'(x) < 0$. Show that $f(x)$ is decreasing on I .

Solution: Let x_1 and x_2 be any two distincts numbers in I such that $x_1 < x_2$. Since $f'(x)$ exists on I , $f(x)$ is differentiable on I and thus it is continuous on I . Therefore, we have that $f(x)$ is continuous on $[x_1, x_2]$ and differentiable on (x_1, x_2) . So by the Mean Value Theorem, there exists c between x_1 and x_2 such that

$$f(x_2) - f(x_1) = f'(c)(x_2 - x_1).$$

Since $f'(c) < 0$ and $x_2 - x_1 > 0$, we conclude that $f(x_2) - f(x_1) < 0$ and, hence, $f(x_2) < f(x_1)$. Thus, $f(x)$ is decreasing on I .

5. The graph of the function $f(x) = -\frac{8}{x^2 - 4}$ is given below. Use this graph to gather information and fill in the blanks. (If a feature doesn't apply, write "None.")



- [1] (a) Equation(s) of any vertical asymptotes: $x = 2, x = -2$
- [1] (b) Equation(s) of any horizontal asymptotes: $y = 0$
- [1] (c) Open intervals where f is increasing: $(0, 2), (2, \infty)$
- [1] (d) Open intervals where f is decreasing: $(-\infty, -2), (-2, 0)$
- [1] (e) x and y -coordinates of any local maxima: None
- [1] (f) x and y -coordinates of any local minima: $(0, 2)$
- [1] (g) Open intervals where f is concave up: $(-2, 2)$
- [1] (h) Open intervals where f is concave down: $(-\infty, -2), (2, \infty)$
- [1] (i) x and y -coordinates of any inflection point(s): None

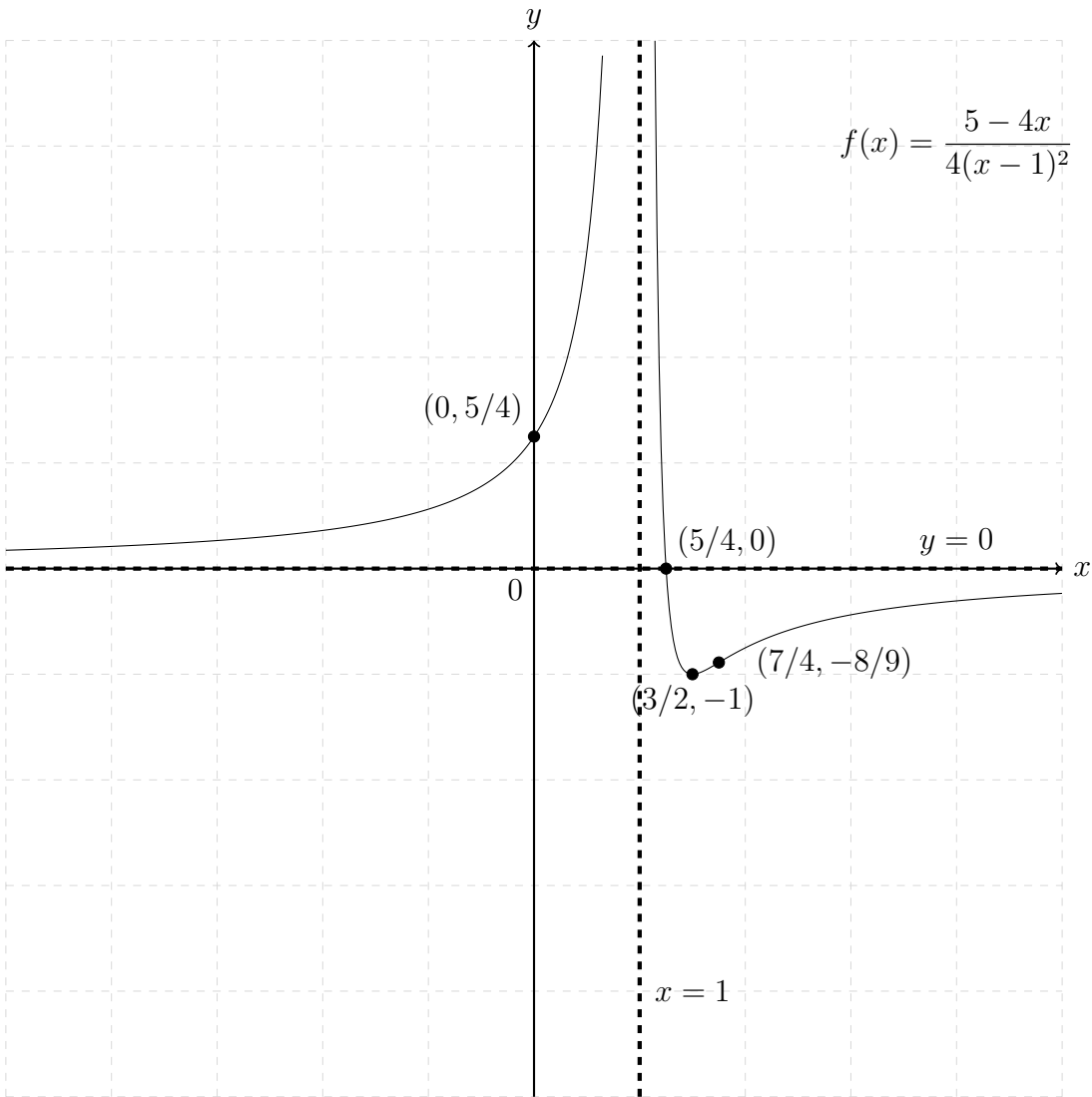
6. Use the function $f(x)$, the first derivative $f'(x)$ and the second derivative $f''(x)$ as defined here to gather information and fill in the blanks below. (If a feature doesn't apply, write "None.")

$f(x) = \frac{5 - 4x}{4(x - 1)^2}$

$f'(x) = \frac{2x - 3}{2(x - 1)^3}$

$f''(x) = \frac{7 - 4x}{2(x - 1)^4}$

- [1] (a) Domain of f : $(-\infty, 1) \cup (1, \infty)$
- [1] (b) Symmetry of f : None
- [1] (c) x -intercepts: $5/4$
- [1] (d) y -intercept: $5/4$
- [1] (e) Equation(s) of any vertical asymptotes: $x = 1$
- [1] (f) Equation(s) of any horizontal asymptotes: $y = 0$
- [1] (g) x and y -coordinates of any critical point(s): $(3/2, -1)$
- [2] (h) Open intervals where f is increasing: $(-\infty, 1), (3/2, \infty)$
- [2] (i) Open intervals where f is decreasing: $(1, 3/2)$
- [1] (j) x and y -coordinates of any local maxima: None
- [1] (k) x and y -coordinates of any local minima: $(3/2, -1)$
- [2] (l) Open intervals where f is concave up: $(-\infty, 1), (1, 7/4)$
- [2] (m) Open intervals where f is concave down: $(7/4, \infty)$
- [1] (n) x and y -coordinates of any inflection point(s): $(7/4, -8/9)$
- [4] (o) Use the information from the previous parts to give a neat sketch of the graph $y = f(x)$, making sure that you label all important features of the graph.



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Term Test 3A

COURSE: MATH 1500

DATE & TIME: November 26, 2018, 5:40PM – 6:40PM

CRN: various

DURATION: 1 hour

EXAMINER: various

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