UNIVERSITY OF MANITOBA
Term Test 2A
COURSE: MATH 1500
DATE & TIME: November 1, 2018, 5:40PM – 6:40PM
CRN: various
DURATION: 1 hour EXAMINER: various

INSTRUCTIONS

I. No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

II. This exam has a title page, 5 pages of questions and two blank pages for rough work. Please check that you have all the pages. **DO NOT REMOVE THE SCRAP PAPER**

III. The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 45 points.

IV. **Answer all questions on the exam paper** in the space provided beneath the question. Unjustified answers will receive little or no credit. Only techniques taught in this course should be used. **Do not continue on the back of the page.** If you need more space, continue on one of the scrap pages, CLEARLY INDICATING THAT YOUR WORK IS TO BE CONTINUED.

V. Do not deface the QR - code in the top right corner. Doing so may result in the page not being scanned and therefore not graded.

Question	Points	Score
1	12	
2	7	
3	5	
4	4	
5	8	
6	9	
Total:	45	

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1. Find the derivative of each of the following functions. DO NOT SIMPLIFY YOUR ANSWER AFTER YOU EVALUATE THE DERIVATIVE.

[4] (a) $f(x) = e^{4x} - 7x^{3/7} + (5\pi)^{-3} + \dfrac{2}{x}$.

Solution: $f'(x) = 4e^{4x} - 3x^{-\frac{4}{7}} + 0 - 2x^{-2}$.

[4] (b) $g(u) = 3^u(\tan u + \ln u)$.

Solution: $g'(u) = 3^u \ln 3(\tan u + \ln u) + 3^u \left(\sec^2 u + \dfrac{1}{u}\right)$.

[4] (c) $h(z) = \dfrac{(2z-4)^3}{1-5z^2}$.

Solution:

$$\begin{aligned} h'(z) &= \frac{((2z-4)^3)'\,(1-5z^2) - (2z-4)^3(1-5z^2)'}{(1-5z^2)^2} \\ &= \frac{(3)(2z-4)^2(2)\,(1-5z^2) - (2z-4)^3(-10z)}{(1-5z^2)^2}. \end{aligned}$$

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[7] 2. Find the derivative of the following function. Express your final answer as a function of only the variable x. DO NOT SIMPLIFY YOUR ANSWER.

$$f(x) = (x+1)^{\cos x}.$$

Solution: Let $y = (x+1)^{\cos x}$. Using logarithmic differentiation, we first take the logarithm of both sides and apply the properties of logarithms before taking the derivative.

$$\begin{aligned} \ln y &= \ln(x+1)^{\cos x} \\ \ln y &= (\cos x)\ln(x+1). \end{aligned}$$

Next, we differentiate both sides with respect to x and then solve for $\frac{dy}{dx}$:

$$\begin{aligned} \frac{1}{y}\frac{dy}{dx} &= (-\sin x)\ln(x+1) + (\cos x)\left(\frac{1}{x+1}\right) \\ \frac{dy}{dx} &= y\left((-\sin x)\ln(x+1) + \frac{\cos x}{x+1}\right) \\ \frac{dy}{dx} &= (x+1)^{\cos x}\left((-\sin x)\ln(x+1) + \frac{\cos x}{x+1}\right). \end{aligned}$$

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3. A particle is moving along the x-axis, and its displacement in meters after t seconds is given by the function $x(t) = t^2 - 5t + 1$.

[2] (a) What are the velocity and acceleration of the particle at any time?

Solution:
Velocity, $v(t) = x'(t) = 2t - 5$ m/s.
Acceleration, $a(t) = x''(t) = 2$ m/s^2.

[3] (b) When is the speed of the particle equal to 1 m/s?

Solution:
The speed of the particle is equal to 1 m/s when,
$|v| = 1 \Longrightarrow |2t - 5| = 1 \Longrightarrow 2t - 5 = 1 \quad \text{or} \quad 2t - 5 = -1 \Longrightarrow t = 3 \quad \text{or} \quad t = 2.$
So, the speed of the particle is equal to 1 m/s when $t = 2$ s and $t = 3$ s.

4. Calculate each limit below. If a limit does not exist, explain why. Show all work. Writing an answer with no justification may not yield any marks.

[1] (a) $\lim\limits_{x\to 0^+} 5\log_2 x$.

Solution: Since $\lim\limits_{x\to 0^+} \log_2 x = -\infty$ then $\lim\limits_{x\to 0^+} 5\log_2 x = -\infty$.

[3] (b) $\lim\limits_{x\to 0} \dfrac{\sin(4x)}{\sin(5x)}$.

Solution:

$$\begin{aligned}
\lim_{x\to 0} \frac{\sin(4x)}{\sin(5x)} &= \lim_{x\to 0} \frac{\sin(4x)}{4x}\left(\frac{5x}{\sin(5x)}\right)\left(\frac{4x}{5x}\right) \\
&= \lim_{x\to 0} \frac{\sin(4x)}{4x} \lim_{x\to 0}\left(\frac{5x}{\sin(5x)}\right) \lim_{x\to 0}\left(\frac{4x}{5x}\right) \\
&= (1)(1)(4/5) = \frac{4}{5}.
\end{aligned}$$

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[8] 5. Find an equation of the tangent to the curve defined implicitly by

$$x^3 + x^2y + y^3 = -8$$

at the point $(2, -2)$.

Solution:

Differentiating the given equation with respect to x, we get

$$\begin{aligned} \frac{d}{dx}(x^3 + x^2y + y^3) &= \frac{d}{dx}(-8) \\ 3x^2 + 2xy + x^2\frac{dy}{dx} + 3y^2\frac{dy}{dx} &= 0 \\ \frac{dy}{dx} &= \frac{-3x^2 - 2xy}{x^2 + 3y^2} \end{aligned}$$

Let m be the value of $\frac{dy}{dx}$ when $x = 2$ and $y = -2$, then $m = -\frac{1}{4}$.
Therefore, an equation of the tangent is

$$y + 2 = -\frac{1}{4}(x - 2).$$

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[9] 6. A rectangular box is designed so that its length is always exactly twice its height. At a particular moment, the box has length 12 cm, width 4 cm and height 6 cm. At this moment, the length is increasing at 3 cm/s. What is the rate of change of width at that moment, if the volume of the box is increasing at 200 cm^3/s? State your final answer in a complete sentence.

Solution: Let l be the length, w the width and h the height. (The students should draw a diagram and label it)
We have $l = 2h$ at all times.
Thus, the volume V is given by

$$V = lwh = \frac{wl^2}{2}.$$

We know that $\frac{dV}{dt} = 200$ cm^3/s and $\frac{dl}{dt} = 3$ cm/s.
We want to find $\frac{dw}{dt}$ when $l = 12$, $w = 4$, and $h = 6$.
Differentiating gives

$$\frac{dV}{dt} = wl\frac{dl}{dt} + \frac{l^2}{2}\frac{dw}{dt}.$$

Replacing by the given values,

$$200 = (4)(12)3 + \frac{12^2}{2}\frac{dw}{dt} \implies \frac{200-144}{72} = \frac{dw}{dt} \implies \frac{dw}{dt} = \frac{7}{9}.$$

Therefore, at the particular moment in question, the width of the box is increasing at the rate of $\frac{7}{9}$ cm/s.

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