UNIVERSITY OF MANITOBA
Term Test 1D
COURSE: MATH 1500
DATE & TIME: October 9, 2018, 5:40PM – 6:40PM
CRN: various
DURATION: 1 hour EXAMINER: various

INSTRUCTIONS

I. No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

II. This exam has a title page, 6 pages of questions and two blank pages for rough work. Please check that you have all the pages. **DO NOT REMOVE THE SCRAP PAPER**

III. The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 50 points.

IV. **Answer all questions on the exam paper** in the space provided beneath the question. Unjustified answers will receive little or no credit. **Do not continue on the back of the page.** If you need more space, continue on one of the scrap pages, CLEARLY INDICATING THAT YOUR WORK IS TO BE CONTINUED.

V. Do not deface the QR - code in the top right corner. Doing so may result in the page not being scanned and therefore not graded.

Question	Points	Score
1	20	
2	7	
3	7	
4	5	
5	5	
6	6	
Total:	50	

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1. Calculate each limit below, if it exists. If a limit does not exist, explain why. Show all work. Writing an answer with no justification may not yield any marks.

[3] (a) $\lim_{x\to 3^+} \dfrac{x^2-7}{|x-3|}$.

> **Solution:** $\lim_{x\to 3^+} x^2-7 = 2$ and $\lim_{x\to 3^+} |x-3| = \lim_{x\to 3^+} (x-3) = 0^+$.
> Therefore the limit goes to either $\pm\infty$.
> Since the fraction is positive, $\lim_{x\to 3^+} f(x) = \infty$.

[5] (b) $\lim_{x\to 4} \dfrac{\sqrt{x+4}-\sqrt{2x}}{x-4}$.

> **Solution:** We rationalize the numerator:
>
> $$\begin{aligned}\lim_{x\to 4} \frac{\sqrt{x+4}-\sqrt{2x}}{x-4} &= \lim_{x\to 4} \frac{(\sqrt{x+4}-\sqrt{2x})(\sqrt{x+4}+\sqrt{2x})}{(x-4)(\sqrt{x+4}+\sqrt{2x})} \\ &= \lim_{x\to 4} \frac{x+4-2x}{(x-4)(\sqrt{x+4}+\sqrt{2x})} \\ &= \lim_{x\to 4} \frac{-(x-4)}{(x-4)(\sqrt{x+4}+\sqrt{2x})} \\ &= \lim_{x\to 4} \frac{-1}{\sqrt{x+4}+\sqrt{2x}} \\ &= -\frac{1}{2\sqrt{8}}.\end{aligned}$$

[6] (c) $\lim_{x\to -\infty} \dfrac{\sqrt{4x^2-9}}{x-5}$.

> **Solution:** Notice that $\sqrt{x^2} = -x$ because $x < 0$.
>
> $$\begin{aligned}\lim_{x\to-\infty} \frac{\sqrt{4x^2-9}}{x-5} &= \lim_{x\to-\infty} \frac{\sqrt{x^2(4-\frac{9}{x^2})}}{x-5} = \lim_{x\to-\infty} \frac{\sqrt{x^2}\sqrt{(4-\frac{9}{x^2})}}{x(1-\frac{5}{x})} \\ &= \lim_{x\to-\infty} \frac{-x\sqrt{(4-\frac{9}{x^2})}}{x(1-\frac{5}{x})} = \lim_{x\to-\infty} \frac{-\sqrt{(4-\frac{9}{x^2})}}{(1-\frac{5}{x})} \\ &= \frac{-\sqrt{(4-0)}}{(1-0)} = -2.\end{aligned}$$

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[2] (d) $\lim_{x\to 1} \dfrac{x^2-1}{x^2+2x-3}$.

Solution:

$$\lim_{x\to 1} \frac{x^2-1}{x^2+2x-3} = \lim_{x\to 1} \frac{(x-1)(x+1)}{(x+3)(x-1)} = \lim_{x\to 1} \frac{(x+1)}{(x+3)} = \frac{1}{2}$$

[4] (e) $\lim_{x\to 4^+} (x-4)^4 \cos\left(\dfrac{5}{\sqrt{x-4}}\right)$.

Solution: For all $x > 4$, $-1 \le \cos\left(\dfrac{5}{\sqrt{x-4}}\right) \le 1$.

Thus, for $x > 4$,

$$-(x-4)^4 \le (x-4)^4 \cos\left(\frac{5}{\sqrt{x-4}}\right) \le (x-4)^4.$$

Since both $(x-4)^4 \to 0$ and $-(x-4)^4 \to 0$, as $x \to 4^+$ then by the Squeeze Theorem,

$$\lim_{x\to 4^+} (x-4)^4 \cos\left(\frac{5}{\sqrt{x-4}}\right) = 0.$$

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[7] 2. Let f be the function:

$$f(x) = \begin{cases} 2c^2x & \text{if } x < 2 \\ 1 & \text{if } x = 2 \\ \left(2c - \frac{1}{2}\right)x & \text{if } x > 2 \end{cases}.$$

Find all values of c for which $f(x)$ is continuous for all real numbers. Be sure to fully justify your answer.

Solution: For any c, the function is continuous on $(-\infty, 2) \cup (2, \infty)$ because it is a polynomial in those intervals.
The function will be continuous at $x = 2$ if

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2) = 1.$$

Furthermore,

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} 2c^2x = 4c^2. \qquad \lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \left(2c - \frac{1}{2}\right)x = 4c - 1.$$

Then,

$$4c^2 = 1 = 4c - 1.$$

We now find c:

$$4c^2 = 1, \quad c = \pm\sqrt{\frac{1}{4}}, \quad c = \pm\frac{1}{2} \qquad\qquad 4c - 1 = 1, \quad 4c = 2, \quad c = \frac{1}{2}.$$

The only value of c that makes both above statements true is $c = \frac{1}{2}$. We conclude that for $c = \frac{1}{2}$ the function $f(x)$ is continuous for all real numbers.

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[7] 3. Use the **definition of derivative** to find $f'(x)$. No credit will be given for any other method.

$$f(x) = \frac{1}{x-4}$$

Solution:

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0} \frac{\dfrac{1}{(x+h)-4} - \dfrac{1}{x-4}}{h} \\
&= \lim_{h\to 0} \frac{x-4-((x+h)-4)}{h((x+h)-4)(x-4)} \\
&= \lim_{h\to 0} \frac{x-4-x-h+4}{h(x+h-4)(x-4)} \\
&= \lim_{h\to 0} \frac{-h}{h(x+h-4)(x-4)} \\
&= \lim_{h\to 0} \frac{-1}{(x+h-4)(x-4)} \\
&= -\frac{1}{(x-4)^2}.
\end{aligned}$$

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[5] 4. Find the domain of the function $f(x) = \sqrt{\dfrac{x+5}{(1-x^2)}}$. Express your final answer in terms of intervals.

Solution: $f(x) = \sqrt{\dfrac{x+5}{(1-x^2)}} = \sqrt{\dfrac{(x+5)}{(1-x)(1+x)}}$.
The radicand needs to be non-negative under the square root and non-zero in the denominator. Let $h(x) = \dfrac{(x+5)}{(1-x)(1+x)}$

x	$x < -5$	$-5 < x < -1$	$-1 < x < 1$	$x > 1$
$1-x$	$+$	$+$	$+$	$-$
$1+x$	$-$	$-$	$+$	$+$
$x+5$	$-$	$+$	$+$	$+$
$h(x)$	$+$	$-$	$+$	$-$

Therefore, $D_f = (-\infty, -5] \cup (-1, 1)$.

[5] 5. Use the Intermediate Value Theorem to show that $4x^3 + 2 - \dfrac{x+1}{x-1} = 0$ has at least one solution on the interval $[-1, 0]$.

Solution: Let $f(x) = 4x^3 + 2 - \dfrac{x+1}{x-1}$, then $f(x)$ is continuous for all x except $x = 1$.
In particular, $f(x)$ is continuous on the interval $[-1, 0]$.
We now check the value of $f(x)$ at the end points:

$$f(-1) = 4(-1)^3 + 2 - \left(\frac{-1+1}{-1-1}\right) = -2 < 0, \quad f(0) = 4(0)^3 + 2 - \left(\frac{0+1}{0-1}\right) = 3 > 0$$

Since the values of the function at the end points have different signs, using the Intermediate Value Theorem, we conclude that there is a point c on the interval $[-1, 0]$ such that $f(c) = 0$. Therefore, there exist at least one solution of the equation on $[-1, 0]$.

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[6] 6. Use the **definition of derivative** to show that

$$f(x) = \begin{cases} x^2 + 2 & \text{if } x \leq 1 \\ 2x + 1 & \text{if } x > 1 \end{cases}$$

is differentiable at $x = 1$.

Solution:

The right-hand limit of the difference quotient of f at $x = 1$ is given by

$$\lim_{h \to 0^+} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0^+} \frac{[2(1+h)+1] - 3}{h} = \lim_{h \to 0^+} \frac{2h}{h} = 2.$$

The left-hand limit of the difference quotient of f at $x = 1$ is given by

$$\lim_{h \to 0^-} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0^-} \frac{[(1+h)^2+2] - 3}{h} = \lim_{h \to 0^-} (2+h) = 2.$$

Since the left-hand and right-hand limits of the difference quotient are the same at $x = 1$, the function f is differentiable there.

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