Summation—Definition

Let $a(x)$ be a function defined at least for positive natural numbers, more generally for integers or even for real numbers. For an integer k we often write a_k for $a(x)$.

$$\sum_{i=m}^{n} a(i) = \sum_{i=m}^{n} a_i = a_m + a_{m+1} + \cdots + a_{n-1} + a_n$$

The symbol $\sum$ is the (upper case) Greek letter *sigma* , i is an index which counts through integers one by one, and $m \leq n$ are integers, the *lower bound* add the $upperbound$ of the summation.

Commonly, but not always, the lower bound is either 0 or 1 .

Summation (Formulas, 1)

$$\begin{aligned}
\sum_{i=1}^{n} k\, a_i &= k \sum_{i=1}^{n} a_i \\
\sum_{i=1}^{n} (a_i + b_i) &= \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i \\
\sum_{i=1}^{n} a_i &= \sum_{i=0}^{n-1} a_{i+1} \\
\sum_{i=1}^{n} a_{i+k} &= \sum_{i=k+1}^{n+k} a_i
\end{aligned}$$

Summation (Formulas, 2)

$$\begin{aligned}
\sum_{i=1}^{n} 1 &= n \\
\sum_{i=1}^{n} i &= \frac{n(n+1)}{2} \\
\sum_{i=1}^{n} i^2 &= \frac{n(n+1)(2n+1)}{6} \\
\sum_{i=1}^{n} i^3 &= \frac{n^2(n+1)^2}{4} \\
\sum_{i=0}^{n} r^i &= \frac{r^{n+1}-1}{r-1} \quad (r \neq 1)
\end{aligned}$$