11 28/ 1

How do we prove the Fundamental Theorem?

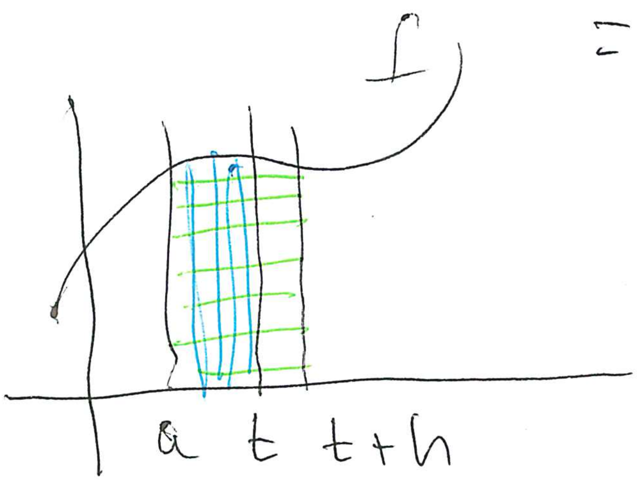
It is just a definition of derivative problem.

By def'n, for t in $[a,b]$,

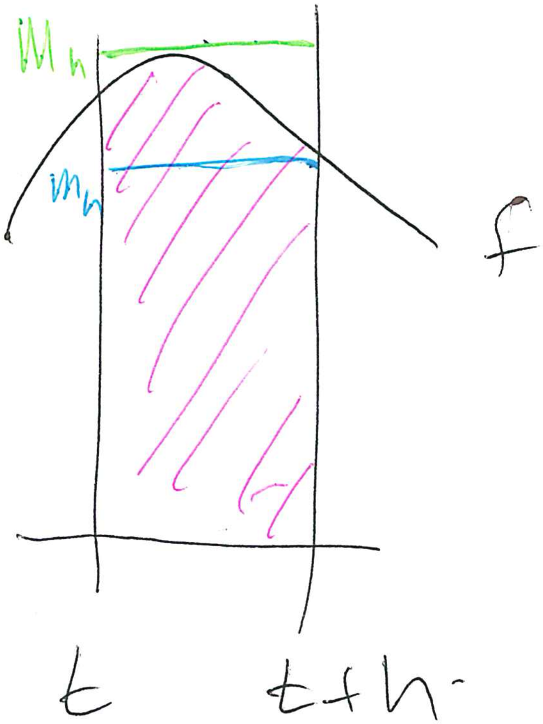
$$F'(t) = \lim_{h \to 0} \frac{F(t+h) - F(t)}{h}$$

$$= \lim_{h \to 0} \frac{A_a^{t+h} f - A_a^{t} f}{h}$$

$$= \lim_{h \to 0} \frac{1}{h} A_t^{t+h} f.$$



11/28/2



Since f is continuous on $[a,b]$

By the extreme value theorem

f has an absolute maximum value M_h on $[t, t+h]$

and an absolute minimum value m_h on $[t, t+h]$

The ~~region~~ area $A_t^{t+h} f$

is trapped between two rectangles

$A_t^{t+h} f$

Therefore $h\, m_h \le A_t^{t+h} f \le h\, M_h$

$$\therefore \quad m_h \le \frac{1}{h} A_t^{t+h} f \le M_h$$

Since f is continuous,

$$\lim_{h\to 0} m_h = f(t) = \lim_{h\to 0} M_h.$$

$\therefore$ By the Squeeze Theorem, $\lim_{h\to 0} \frac{1}{h} A_t^{t+h} f = f(t)$

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