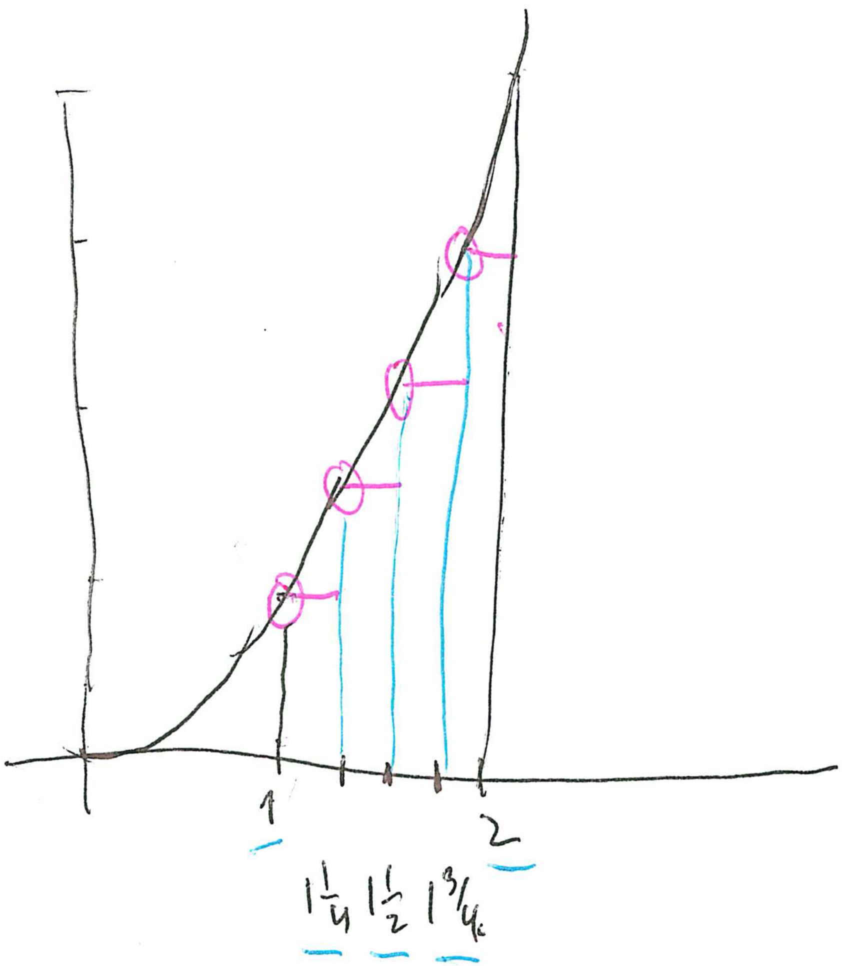
Let's look at $f(x) = x^2$ on $[1, 2]$



Take a partition into four equal subintervals

$x_0 = 1$, $x_1 = 1\frac{1}{4}$, $x_2 = 1\frac{1}{2}$, $x_3 = 1\frac{3}{4}$, $x_4 = 2$

Take as sample points the left hand endpoints

$x_1^* = 1$, $x_2^* = 1\frac{1}{4}$, $x_3^* = 1\frac{1}{2}$, $x_4^* = 1\frac{3}{4}$.

The associated Riemann sum is $\frac{1}{4}(1)^2 + \frac{1}{4}\left(\frac{5}{4}\right)^2 + \frac{1}{4}\left(\frac{3}{2}\right)^2 + \frac{1}{4}\left(\frac{7}{4}\right)^2$.

$= \frac{1}{4}(1 + 1.5625 + 2.25 + 3.0625)$

$= \frac{1}{4}(7.885) \approx 1.97.$

We do the same thing for general n.

If we divide $[1,2]$ into n equal parts the length of each subinterval Δx is $\frac{1}{n}$ and the points of the partition are $x_0 = 1$, $x_1 = 1+\frac{1}{n}$, $x_2 = 1+\frac{2}{n}$, $\ldots$ $x_{n-1} = 1+\frac{n-1}{n}$, $x_n = 2$.

The associated Riemann sum (left hand endpoints) is

$$\frac{1}{n}(1)^2 + \frac{1}{n}\left(1+\frac{1}{n}\right)^2 + \frac{1}{n}\left(1+\frac{2}{n}\right)^2 + \ldots + \frac{1}{n}\left(1+\frac{n-1}{n}\right)^2$$

$$= \frac{1}{n}\left[1^2 + \left(1+\frac{1}{n}\right)^2 + \left(1+\frac{2}{n}\right)^2 + \ldots + \left(1+\frac{n-1}{n}\right)^2\right]$$

$$= \frac{1}{n}\left[(1^2+0+0) + \left(1+\frac{2}{n}+\frac{1}{n^2}\right) + \left(1+\frac{4}{n}+\frac{4}{n^2}\right) + \ldots + \left(1+2\frac{(n-1)}{n}+\left(\frac{n-1}{n}\right)^2\right)\right]$$

$$= \frac{1}{n}\left[n + \sum_{i=0}^{n-1} 2\left(\frac{i}{n}\right) + \sum_{i=0}^{n-1}\left(\frac{i}{n}\right)^2\right]$$

$$= \frac{1}{n}\left[n + \frac{2}{n}\sum_{i=0}^{n-1} i + \frac{1}{n^2}\sum_{i=0}^{n-1} (i)^2\right]$$

$$= \frac{1}{n}\left[n + \frac{2}{n}\frac{(n-1)n}{2} + \frac{1}{n^2}\left[\frac{(n-1)n(2n-1)}{6}\right]\right] = \frac{14n^2-9n+1}{6n^2}$$

$$= \frac{7}{3} - \frac{3}{2n} - \frac{1}{6n^2}.$$ Taking the limit as $n \to \infty$

we get: $\int_1^2 x^2\,dx = \frac{7}{3}$.