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General patterns:

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x).$$

$$\frac{d}{dx}\int_x^b f(t)\,dt = -f(x)$$

$$\frac{d}{dx}\int_a^{b(x)} f(t)\,dt = f(b(x))\,b'(x)$$

$$\frac{d}{dx}\int_{a(x)}^{b} f(t)\,dt = -f(a(x))\,a'(x)$$

$$\frac{d}{dx}\int_{a(x)}^{b(x)} f(t)\,dt = f(b(x))\,b'(x) - f(a(x))\,a'(x).$$

Fundamental Theorem Part 2

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

(where F is an antiderivative of f)

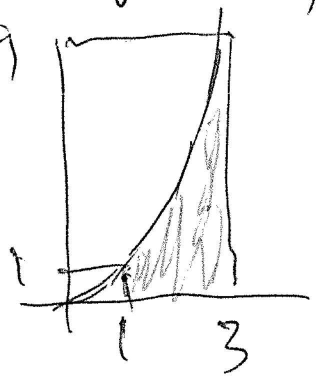
Evaluation notation.

If F is any function defined on an interval containing a and b,

then $$F\Big|_a^b = F(b) - F(a)$$

Eg $$\int_1^3 x^2\,dx = \frac{x^3}{3}\Big|_1^3 = \frac{27}{3} - \frac{1}{3} = \frac{26}{3}$$

Ex Find the area under the parabola $y = x^2$, above the x-axis, between $x=1$ and $x=3$



Ans $\int_1^3 x^2\,dx$ = (already solved)

Ex $\displaystyle\int_1^4 \frac{dx}{x^2} \left(= \int_1^4 \frac{1}{x^2}\,dx \right) \left(= \int_1^4 x^{-2}\,dx \right)$

$$= \left. \frac{x^{-1}}{-1} \right|_1^4 = -\frac{1}{4} - (-1) = 3/4.$$

$\displaystyle\int_{-1}^{1} \frac{dx}{x^2}.$ $\quad \frac{1}{x^2}$ is not defined on all of $[-1,1]$

So the definite integral is not defined.

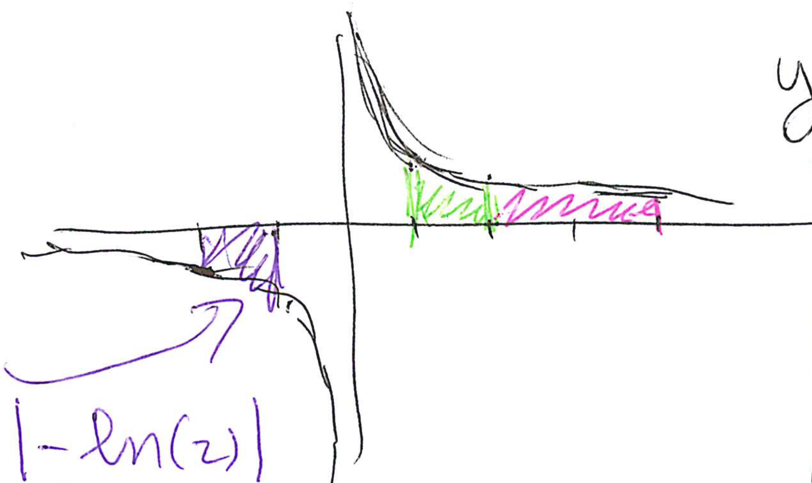
Ex $\int_1^2 \frac{dx}{x} = \ln(x)\Big|_1^2 \qquad (x>0)$ 1203/4

$$= \ln(2) - \ln(1) = \ln(2)$$

$$\int_2^4 \frac{dx}{x} = \ln(x)\Big|_2^4 = \ln(4) - \ln(2) = \ln\left(\frac{4}{2}\right) = \ln(2)$$

$$\int_a^{2a} \frac{dx}{x} = \ln(x)\Big|_a^{2a} = \ln(2a) - \ln(a) = \ln\left(\frac{2a}{a}\right) = \ln 2$$

$$\int_{-2}^{-1} \frac{dx}{x} = \ln|x|\Big|_{-2}^{-1} = \ln(1) - \ln(2) = -\ln(2)$$

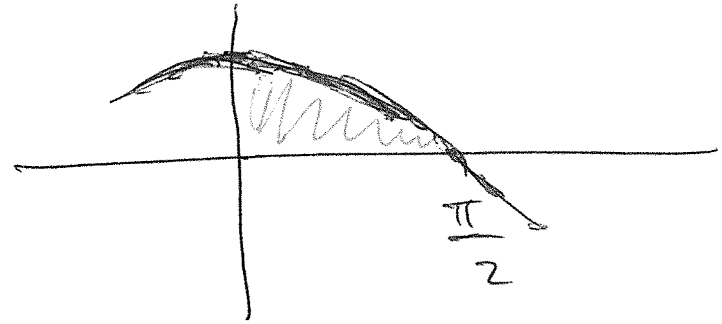


$y = \frac{1}{x}$

area $= |-\ln(2)|$
$= \ln(2)$

What if the question were "What is the area enclosed by $y = \frac{1}{x}$ and the x-axis, between $x=-2$ and $x=-1$?

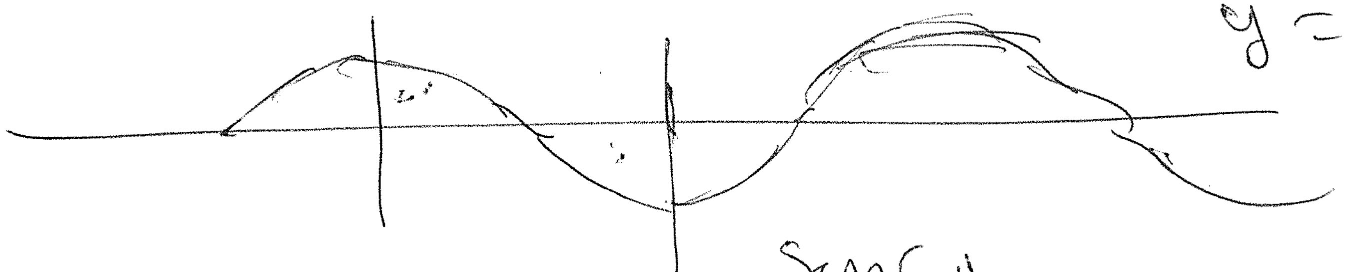
Ex $\int_0^{\pi/2} \cos x \, dx$



$$= \sin(x)\Big|_0^{\pi/2} = \sin\left(\frac{\pi}{2}\right) - \sin(0) = 1$$

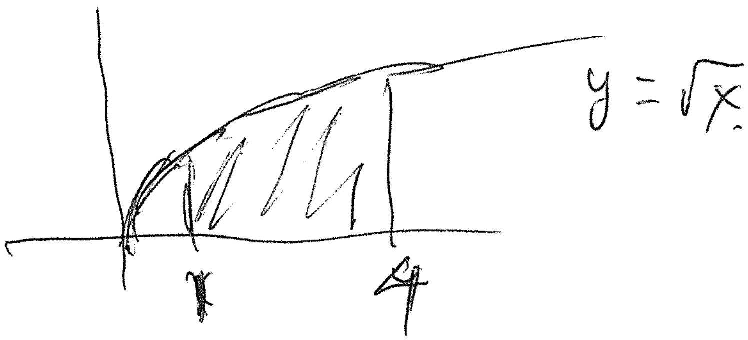
Ex $\int_0^{\theta} \cos(x)dx = \sin(x)\Big|_0^{\theta} = \sin(\theta)$

$y = \cos x$



$\sin(\pi) = 0$.

Ex Find the area under $y = \sqrt{x}$ between $x = 1$ and $x = 4$.



$$\text{Area} = \int_1^4 x^{1/2}\,dx.$$

$$= \left. \frac{x^{3/2}}{3/2} \right|_1^4.$$

$$= \tfrac{2}{3}(8) - \tfrac{2}{3}(1) = \frac{14}{3}.$$

Ex Find the are-

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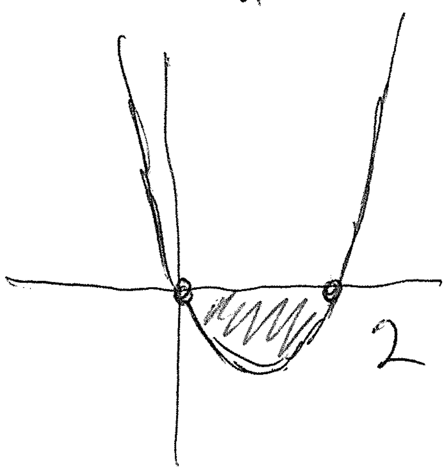
__Ex__ Find the area of the region enclosed by $y = x(x-2)$ and the x-axis.



$$y = x(x-2)$$
$$= x^2 - 2x$$

The region lies below the x-axis

$$\text{Area} = \left| \int_0^2 x^2 - 2x \, dx \right|$$

$$= \left| \frac{x^3}{3} - \cancel{2}\frac{x^2}{\cancel{2}} \Big|_0^2 \right|$$

$$= \left| \left(\frac{8}{3} - 4\right) - 0 \right| = \left| -\frac{4}{3} \right| = \frac{4}{3}$$

Alternate.

$\int_0^2 x^2 - 2x \, dx = \ldots = -\frac{4}{3}$; Area $= \left| -\frac{4}{3} \right| = 4/3$

Section 5.4 Indefinite Integrals

Net change Theorem

Antiderivative =

most general antiderivative
"indefinite integral"

$$\int f(x)\,dx$$

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Ex $\int \frac{x^3 + \sqrt{x} + 5}{x^2}\, dx$

$= \int (x + x^{-3/2} + 5x^{-2})\, dx$

$= \frac{x^2}{2} + \frac{x^{-1/2}}{-1/2} + 5\frac{x^{-1}}{-1} + C.$

Ex 2 p409. $\int \frac{\cos\theta}{\sin^2\theta}\, d\theta = \int \frac{1}{\sin\theta}\frac{\cos\theta}{\sin\theta}\, d\theta$

$= \int \csc\theta \cot\theta = -\csc\theta\, d\theta$

The Net Change Theorem.

Suppose $y = f(t)$ represents a rate of change (of something)

Then the antiderivative $F(t)$ ~~repres~~ represents the thing changing

So $\int_a^b f(t)\,dt = F(b) - F(a)$

says that "The definite integral of a rate of change over an interval represents the net change in the quantity that is varying

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