1/23/7

"Initial value problems"

Ex Suppose $f'(x) = -x^3 + 3x^2 + 7$ and $f(1) = 1$

Find f.

$$f(x) = \int -x^3 + 3x^2 + 7 \, dx = -\frac{x^4}{4} + 3\frac{x^3}{3} + 7x + C$$

$$= -\frac{1}{4}x^4 + x^3 + 7x + C.$$

$$1 = f(1) = -\frac{1}{4} + 1 + 7 + C, \text{ so } C = -6\tfrac{3}{4}$$

So $f(x) = -\frac{1}{4}x^4 + x^3 + 7x - 6\frac{3}{4}$

Ex Suppose $f'(\theta) = \cos(\theta) - 3\sin\theta$ and $f(0) = 2$

Find $f(\theta)$. $\int \cos(\theta) - 3\sin(\theta)\, d\theta = \sin(\theta) + 3\cos(\theta) + C$

So, $\sin(0) + 3\cos(0) + C = 2$, $+3 + C = 2$

So $C = \cancel{2}\ -1$; $f(\theta) = \sin\theta + 3\cos\theta - 1$

1/23/18

Position/velocity/acceleration problems

t time

position $s(t)$

velocity $v(t) = \frac{d}{dt}s(t)$

acceleration $a(t) = \frac{d}{dt}v(t) = \frac{d^2}{(dt)^2}s(t)$

$$\therefore \quad v(t) = \int a(t)\,dt.$$

$$s(t) = \int v(t)\,dt.$$

11/23/9

Ex A particle moves on a straight line (the x-axis) with acceleration $a(t) = 12t + 3$ (m/sec^2)

At time $t = 0$ the particle is at $x = 5m$ moving to the left at 3 m/sec.

Where is it at time $t = 2$ sec and how fast is it moving?

Given $a(t) = 12t + 3$, $x(0) = 5$, $v(0) = -3$.

* Moving to left = negative velocity

So take antiderivatives:

$v(t) = 12\frac{t^2}{2} + 3t + C$; $-3 = v(0) = C$.

$v(t) = 6t^2 + 3t - 3$.

$x(t) = 6\frac{t^3}{3} + 3\frac{t^2}{2} - 3t + C$; $5 = x(0) = C$.

$x(t) = 2t^3 + \frac{3}{2}t^2 - 3t + 5$

At $t = 2$ the particle is at $x(2) = 16 + 6 - 6 + 5 = 21$ m

moving at $v(t) = 24 + 6 - 3 = 27$ m/sec (to the right).