MATH 1500 (A02) Friday Sept 14

(very much) supplementary notes

Apology: In this lecture I got off track on an example. I had actually meant to use it in a much simpler way than I tried in class; the full "working out" of what I started there is really beyond the scope of the course. But since I did start it in class, I include some details here.

Question: Is $f(x) = x^2$ well behaved near $a = 2$? That is, if we have a good approximation of $x = 2$, does $f(x)$ give us a good approximation of 4?

The intended response: Draw the graph of $y = x^2$. We are very familiar with this well-behaved curve from high-school. Part of this familiarity is that small changes in x produce small changes in y, allowing us to smoothly draw a curve connecting known points on the graph.

The answer is "obviously yes".

The somewhat over-the-top technical answer: What is $\lim_{x \to 2} x^2$?

Based on our understanding of the graph, we know that the answer should be 4. So the questions is "If x is near 2, is x^2 always near 4?

Suppose that t is some small positive real number and we want to force x^2 to be within t of 4, that is, $\left| x^2 - 4 \right| < t$. Can we do this by choosing x sufficiently close to 2?

Here's where things get awkward: we "look ahead" to see how the algebraic manipulations might work out, and "pull a rabbit out of a hat". [This is why I never intended to do this in a lecture!]

Let's force x to be close to 2: We choose

$$\sqrt{4-t} < x < \sqrt{4+t}$$

Then

$$4 - t \leq x^2 < 4 + t$$

So, $\lim_{x \to 2} x^2 = 4$.

Comment Technically, this is not a particularly good way of explaining this either, but the "best way" would take us even further away from what is actually needed for MATH 1500.