MATH 1300(A02) QUIZ 5 April 04, 2016 B08

NAME SOLUTIONS ID Number

INSTRUCTIONS: Answer the following questions in the spaces provided below.

[12] TOTAL

[3] **1.** Suppose that T_1 and T_2 are linear transformations from $\mathbb{R}^n$ to $\mathbb{R}^m$, and define U by $U(\mathbf{x}) = T_1(\mathbf{x}) - T_2(\mathbf{x})$ for all $(\mathbf{x})$ in $\mathbb{R}^n$. Prove that U is a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$.

Let $\underline{x}_1$ and $\underline{x}_2$ be vectors in $\mathbb{R}^n$ and c a scalar

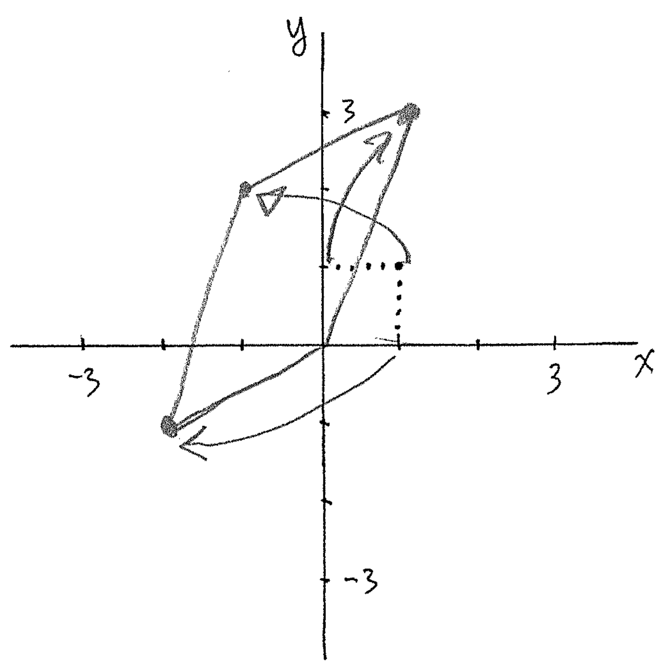
$U(\underline{x}_1+\underline{x}_2) = T_1(\underline{x}_1+\underline{x}_2) - T_2(\underline{x}_1+\underline{x}_2) = T_1(\underline{x}_1) + T_1(\underline{x}_2) - T_2(\underline{x}_1) - T_2(\underline{x}_2)$

$= (T_1(\underline{x}_1) - T_2(\underline{x}_1)) + (T_1(\underline{x}_2) - T_2(\underline{x}_2)) = U(\underline{x}_1) + U(\underline{x}_2)$

$U(c\underline{x}_1) = T_1(c\underline{x}_1) - T_2(c\underline{x}_1) = cT_1(\underline{x}_1) - cT_2(\underline{x}_1)$

$= c(T_1(\underline{x}_1) - T_2(\underline{x}_1)) = c\,U(\underline{x}_1)$

[4] **2.** Let $T_A : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by the matrix $A = \begin{bmatrix} -2 & 1 \\ -1 & 3 \end{bmatrix}$. Give a sketch showing how T_A transforms the unit square.



$A\underline{e}_1 = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$

$A\underline{e}_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$A(\underline{e}_1+\underline{e}_2) = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$.

[5] **3.** Find the eigenvalues and the associated eigenvectors of $A = \begin{bmatrix} -10 & 24 \\ -4 & 10 \end{bmatrix}$.

Solve $\begin{vmatrix} \lambda+10 & -24 \\ 4 & \lambda-10 \end{vmatrix} = 0$.

$0 = (\lambda+10)(\lambda-10) + 96 = \lambda^2 - 100 + 96 = \lambda^2 - 4$

$= (\lambda-2)(\lambda+2)$. Eigenvalues $2, -2$.

For $\lambda = 2$. $12x_1 - 24x_2 = 0$, $x_1 = 2x_2$, so $t\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

For $\lambda = -2$ $8x_1 - 24x_2 = 0$, $x_1 = 3x_2$, so $t\begin{bmatrix} 3 \\ 1 \end{bmatrix}$