MATH 1300(A02) QUIZ 5 April 01, 2016 **B07**

NAME SOLUTIONS. ID Number

INSTRUCTIONS: Answer the following questions in the spaces provided below.

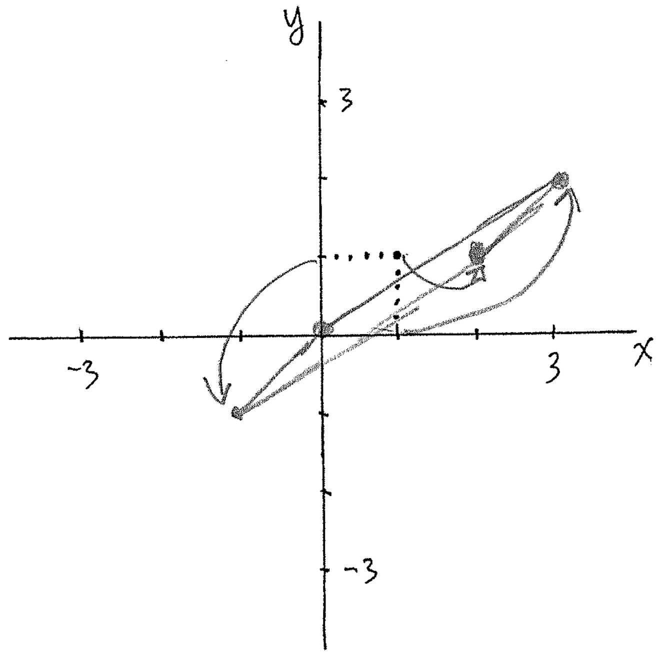
[12] TOTAL

[3] **1.** Suppose that T is a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$ such that $T(\mathbf{e}_1) = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$ and $T(\mathbf{e}_2) = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$. Find the value of $T\left(\begin{bmatrix} 5 \\ 3 \end{bmatrix}\right)$.

$$T\left(\begin{bmatrix} 5 \\ 3 \end{bmatrix}\right) = T(5\underline{e}_1 + 3\underline{e}_2) = 5T(\underline{e}_1) + 3T(\underline{e}_2).$$

$$= 5\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} + 3\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 13 \\ -2 \\ 12 \end{bmatrix}.$$

[4] **2.** Let $T_A : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by the matrix $A = \begin{bmatrix} 3 & -1 \\ 2 & -1 \end{bmatrix}$. Give a sketch showing how T_A transforms the unit square.



$$A\underline{e}_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$A\underline{e}_2 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$A(\underline{e}_1 + \underline{e}_2) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

[5] **3.** Find the eigenvalues and the associated eigenvectors of $A = \begin{bmatrix} 11 & -24 \\ 4 & -9 \end{bmatrix}$.

Solve $\begin{vmatrix} \lambda - 11 & 24 \\ -4 & \lambda + 9 \end{vmatrix} = 0$

$0 = (\lambda - 11)(\lambda + 9) + 96 = \lambda^2 - 2\lambda - 99 + 96 = \lambda^2 - 2\lambda - 3$

$= (\lambda - 3)(\lambda + 1)$ Eigenvalues $3, -1$.

For $\lambda = 3$ $-8x_1 + 24x_2 = 0$, $x_1 = 3x_2$, so $t\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

For $\lambda = -1$, $-12x_1 + 24x_2 = 0$, $x_1 = 2x_2$,

So $t\begin{bmatrix} 2 \\ 1 \end{bmatrix}$