MATH 1300(A02) QUIZ 5 April 01, 2016 **B06**

NAME SOLUTIONS. ID Number

INSTRUCTIONS: Answer the following questions in the spaces provided below.

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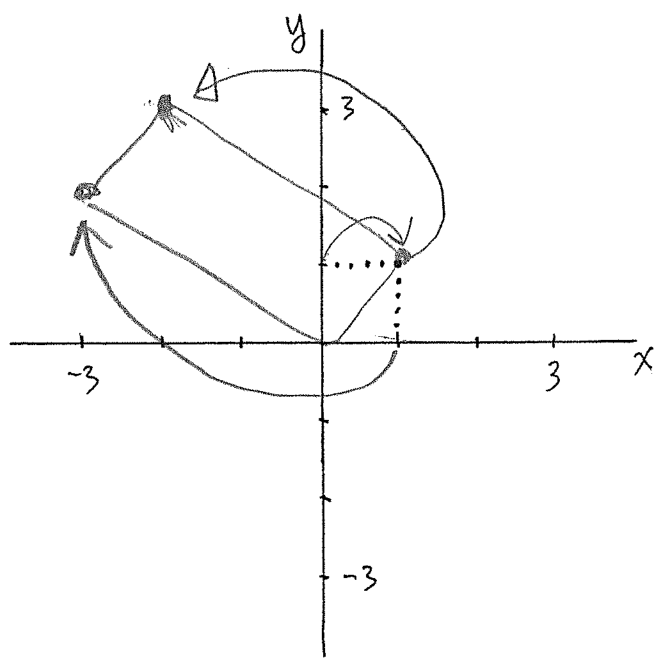
[12] TOTAL

[3] **1.** Write the definition of "T is a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$".

A function T from $\mathbb{R}^n$ to $\mathbb{R}^m$ is a linear transformation if for all $\underline{x}, \underline{y}$ in $\mathbb{R}^n$ and all scalars c,

$$T(\underline{x}+\underline{y}) = T(\underline{x}) + T(\underline{y})$$
$$T(c\underline{y}) = cT(\underline{y})$$

[4] **2.** Let $T_A : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by the matrix $A = \begin{bmatrix} -3 & 1 \\ 2 & 1 \end{bmatrix}$. Give a sketch showing how T_A transforms the unit square.



$$A\underline{e}_1 = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$$

$$A\underline{e}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$A(\underline{e}_1+e_2) = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

[5] **3.** Find the eigenvalues and the associated eigenvectors of $A = \begin{bmatrix} -12 & 30 \\ -5 & 13 \end{bmatrix}$.

Solve $\begin{vmatrix} \lambda+12 & -30 \\ 5 & \lambda-13 \end{vmatrix} = 0$.

$0 = (\lambda+12)(\lambda-13) + 150 = \lambda^2 - \lambda - 156 + 150 = \lambda^2 - \lambda - 6$

$= (\lambda-3)(\lambda+2)$ eigenvalues $3, -2$.

For $\lambda = -3$, $15x_1 - 30x_2 = 0$, $x_1 = 2x_2$, so $t\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

For $\lambda = -2$, $10x_1 - 30x_2 = 0$, $x_1 = 3x_2$, so $t\begin{bmatrix} 3 \\ 1 \end{bmatrix}$