MATH 1300(A02)   QUIZ 5   April 01, 2016   B05

NAME SOLUTIONS. ID Number

INSTRUCTIONS: Answer the following questions in the spaces provided below.

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[12] TOTAL

[3] **1.** Prove that if T and U are linear transformations from $\mathbb{R}^2$ to $\mathbb{R}^4$ satisfying $T(\mathbf{e}_1) = U(\mathbf{e}_1)$ and $T(\mathbf{e}_2) = U(\mathbf{e}_2)$, then $T(\mathbf{x}) = U(\mathbf{x})$ for all $\mathbf{x}$ in $\mathbb{R}^2$.

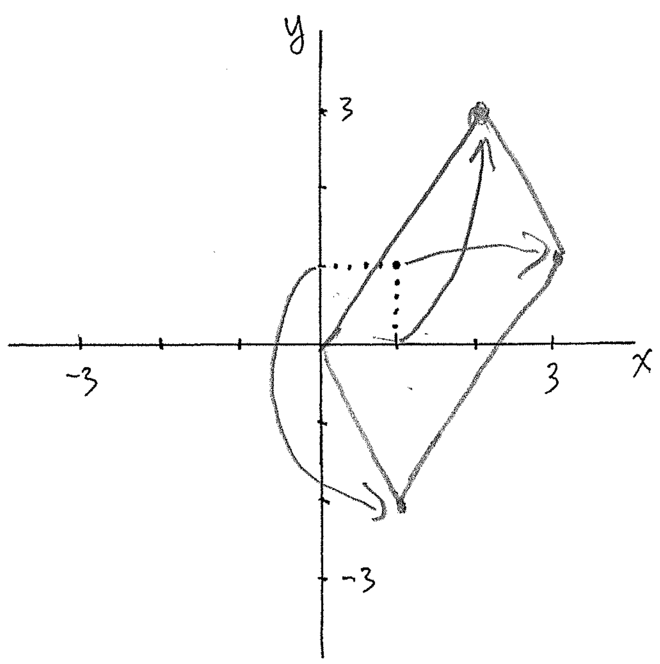
~~Ex~~ If $\underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ then $\underline{x} = x_1 \underline{e}_1 + x_2 \underline{e}_2$.

So $T(\underline{x}) = T(x_1\underline{e}_1 + x_2\underline{e}_2) = x_1 T(\underline{e}_1) + x_2 T(\underline{e}_2)$

$= x_1 U(\underline{e}_1) + x_2 U(\underline{e}_2) = U(x_1\underline{e}_1 + x_2\underline{e}_2)$

$= U(\underline{x})$.

[4] **2.** Let $T_A : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by the matrix $A = \begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix}$. Give a sketch showing how T_A transforms the unit square.



$A(\underline{e}_1) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$A(\underline{e}_2) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$A(\underline{e}_1 + \underline{e}_2) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$.

[5] **3.** Find the eigenvalues and the associated eigenvectors of $A = \begin{bmatrix} 8 & -18 \\ 3 & -7 \end{bmatrix}$.

Solve. $\begin{vmatrix} \lambda - 8 & 18 \\ -3 & \lambda + 7 \end{vmatrix} = 0$.

$0 = (\lambda - 8)(\lambda + 7) + 54 = \lambda^2 - \lambda - 56 + 54 = \lambda^2 - \lambda - 2$

$= (\lambda - 2)(\lambda + 1)$. Eigenvalues $2, -1$.

For $\lambda = 2$, $-6x_1 + 18x_2 = 0$, $x_1 = 3x_2$, so $t\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

For $\lambda = -1$, $-9x_1 + 18x_2 = 0$, $x_1 = 2x_2$, so $t\begin{bmatrix} 2 \\ 1 \end{bmatrix}$.