

NAME

SOLUTIONS

ID Number

INSTRUCTIONS: Answer the following questions in the spaces provided below.

[12] TOTAL

[4] 1.

(a) State the *Cauchy-Schwarz Inequality* for two vectors $\underline{u}$ and $\underline{v}$.

$$|\underline{u} \cdot \underline{v}| \leq \|\underline{u}\| \|\underline{v}\|$$

(b) State the *parallelogram law* for vector norms.

$$\|\underline{u} + \underline{v}\|^2 + \|\underline{u} - \underline{v}\|^2 = 2(\|\underline{u}\|^2 + \|\underline{v}\|^2)$$

[4] 2. Let $\underline{a} = \langle 1, -1, 1, -1 \rangle$ and $\underline{u} = \langle -2, 1, 4, -3 \rangle$.Find the vector component of $\underline{u}$ orthogonal to $\underline{a}$.

$$\begin{aligned} \underline{u} - \text{proj}_{\underline{a}}(\underline{u}) &= \underline{u} - \frac{\underline{u} \cdot \underline{a}}{\|\underline{a}\|^2} \underline{a} \\ &= \langle -2, 1, 4, -3 \rangle - \frac{\langle -2, 1, 4, -3 \rangle \cdot \langle 1, -1, 1, -1 \rangle}{1^2 + 1^2 + 1^2 + 1^2} \langle 1, -1, 1, -1 \rangle \\ &= \langle -2, 1, 4, -3 \rangle - \frac{1}{4}(-2 - 1 + 4 + 3) \langle 1, -1, 1, -1 \rangle \\ &= \langle -2, 1, 4, -3 \rangle - \langle 1, -1, 1, -1 \rangle \\ &= \langle -3, -2, 3, -2 \rangle \end{aligned}$$

[4] 3. Find a vector form of the equation of the plane containing the point $\underline{b} = \langle 3, -3, 0 \rangle$ and the straight line through $\underline{a} = \langle 1, -3, 2 \rangle$ in the direction of $\underline{u} = \langle -1, 0, -1 \rangle$.

So the direction vector of the line is one direction vector of the plane, and the vector from $\underline{a}$ to $\underline{b} = \langle 3, -3, 0 \rangle - \langle 1, -3, 2 \rangle = \underline{v} = \langle 2, 0, -2 \rangle$ is another (or calculate $\underline{a} - \underline{b} = \langle -2, 0, 2 \rangle$.)

So an equation of the plane is

$$\begin{aligned} \underline{x} &= \underline{a} + s \underline{u} + t \underline{v} \\ \text{[or } \underline{x} &= \underline{b} + s \underline{u} + t \underline{v} \text{ if you want.]} \\ \langle x, y, z \rangle &= \langle 1, -3, 2 \rangle + s \langle -1, 0, -1 \rangle + t \langle 2, 0, -2 \rangle \end{aligned}$$