

NAME SOLUTIONS ID Number _____

INSTRUCTIONS: Answer the following questions in the spaces provided below.

[12] TOTAL

[4]

1.

(a) State the formula for the cosine of the angle between two vectors $\underline{u}$ and $\underline{v}$.

$$\cos \theta = \frac{\underline{u} \cdot \underline{v}}{\|\underline{u}\| \|\underline{v}\|}$$

(b) State the *parallelogram law* for vector norms.

$$\|\underline{u} + \underline{v}\|^2 + \|\underline{u} - \underline{v}\|^2 = 2(\|\underline{u}\|^2 + \|\underline{v}\|^2)$$

[4]

2. Let $\underline{a} = \langle -1, 1, 1, -1 \rangle$ and $\underline{u} = \langle 2, -1, 4, -3 \rangle$.Find the vector component of $\underline{u}$ along $\underline{a}$.

$$\text{proj}_{\underline{a}}(\underline{u}) = \frac{\underline{u} \cdot \underline{a}}{\|\underline{a}\|^2} \underline{a}$$

$$= \frac{\langle 2, -1, 4, -3 \rangle \cdot \langle -1, 1, 1, -1 \rangle}{1^2 + 1^2 + 1^2 + 1^2} \langle -1, 1, 1, -1 \rangle$$

$$= \frac{1}{4} (-2 - 1 + 4 + 3) \langle -1, 1, 1, -1 \rangle$$

[4]

3. Find a vector form of the equation of the straight line through the points $\langle 2, -1, 0, 3 \rangle$ and $\langle 0, 2, -3, 1 \rangle$.

We can subtract in either order to get a direction vector $\underline{v}$, say

$$\underline{v} = \langle 2, -1, 0, 3 \rangle - \langle 0, 2, -3, 1 \rangle$$

$$= \langle 2, -3, +3, 2 \rangle$$

and use either point for the equation,

$$\text{Say } \langle x_1, x_2, x_3, x_4 \rangle = \langle 2, -1, 0, 3 \rangle + t \langle 2, -3, 3, 2 \rangle$$