MATH 1300(A02) QUIZ 4 March 18, 2016 **B06**

NAME SOLUTIONS ID Number

INSTRUCTIONS: Answer the following questions in the spaces provided below.

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[12] TOTAL

[4] **1.**
(a) State the *Cauchy-Schwarz Inequality* for two vectors **u** and **v**.

$$|\underline{u} \cdot \underline{v}| \leq \|\underline{u}\| \|\underline{v}\|$$

(b) State the *triangle inequality* for vector norms.

$$\|\underline{u} + \underline{v}\| \leq \|\underline{u}\| + \|\underline{v}\|$$

[4] **2.** Let $\mathbf{a} = \langle -1, -1, 1, 1 \rangle$ and $\mathbf{u} = \langle 2, -1, 4, -3 \rangle$.
Find the vector component of **u** orthogonal to **a**.

$$\underline{u} - \text{proj}_{\underline{a}}(\underline{u}) = \underline{u} - \frac{\underline{u} \cdot \underline{a}}{\|a\|^2}\underline{a}.$$

$$= \langle 2,-1,4,-3\rangle - \frac{\langle 2,-1,4,-3\rangle \cdot \langle -1,-1,1,1\rangle}{(1^2+1^2+1^2+1^2)}\langle -1,-1,1,1\rangle$$

$$= \langle 2,-1,4,-3\rangle - \frac{1}{4}(-2+1+4-3)\langle -1,-1,1,1\rangle$$

$$= \langle 2,-1,4,-3\rangle - \underline{0}.$$

$$= \underline{u}\ .$$

[4] **3.** Find a vector form of the equation of the plane through $\langle 2, -3, 1 \rangle$ with normal vector $\mathbf{n} = \langle 1, 2, 3 \rangle$.

(Point normal form)

$$(\underline{x} - \langle 2,-3,1\rangle) \cdot (1,2,3) = 0$$

$$\langle x-2, y+3, z-1\rangle \cdot \langle 1,2,3\rangle = 0$$

$$(x-2) + 2(y+3) + 3(z-1) = 0$$

$$x + 2y + 3z = -1$$

In augmented matrix form this is

$$[1\ ,\ 2\quad 3 \mid -1]$$

which is already rref.

Take $y = s$, $z = t$ so $x = -1 - 2s - 3t$

$y = s$

$z = t$

or $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} + s\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$