

NAME SOLUTIONS ID Number _____

INSTRUCTIONS: Answer the following questions in the spaces provided below.

[12] TOTAL

[4] 1.

(a) State the formula for the cosine of the angle between two vectors $\underline{u}$ and $\underline{v}$.

$$\cos \theta = \frac{\underline{u} \cdot \underline{v}}{\|\underline{u}\| \|\underline{v}\|}$$

(b) State the *triangle inequality* for vector norms.

$$\|\underline{u} + \underline{v}\| \leq \|\underline{u}\| + \|\underline{v}\|$$

[4] 2. Let $\underline{a} = \langle 1, -1, -1, 1 \rangle$ and $\underline{u} = \langle 2, -1, 4, -3 \rangle$.
Find the vector component of $\underline{u}$ along $\underline{a}$.

$$\begin{aligned} \text{proj}_{\underline{a}}(\underline{u}) &= \frac{\underline{u} \cdot \underline{a}}{\|\underline{a}\|^2} \underline{a} \\ &= \frac{\langle 1, -1, -1, 1 \rangle \cdot \langle 2, -1, 4, -3 \rangle}{(1^2 + 1^2 + 1^2 + 1^2)} \langle 1, -1, -1, 1 \rangle \\ &= \frac{1}{4} (2 + 1 - 4 - 3) \langle 1, -1, -1, 1 \rangle \\ &= \langle -1, 1, 1, -1 \rangle. \end{aligned}$$

[4] 3. Let $\underline{a} = \langle 3, -2 \rangle$ and $\underline{u} = \langle 1, 2 \rangle$.Find an equation of the straight line through $\underline{a}$ in the direction of $\underline{u}$, and find an equation of the straight line through $\underline{a}$ and perpendicular to $\underline{u}$.

- An equation of the line through $\underline{a}$ in the direction of $\underline{u}$ is $\underline{x} = \underline{a} + t \underline{u}$.
or $\langle x, y \rangle = \langle 3 + t, -2 + 2t \rangle$
- An equation of the line through $\underline{a}$ perpendicular to $\underline{u}$ is $(\underline{x} - \underline{a}) \cdot \underline{u} = 0$
or $\langle x - 3, y + 2 \rangle \cdot \langle 1, 2 \rangle = 0$
 $(x - 3) + 2(y + 2) = 0$
 $x + 2y = -1.$