MATH 1300(A02) QUIZ 3 March 07, 2016 B08

NAME SOLUTIONS ID Number

INSTRUCTIONS: Answer the following questions in the spaces provided below.

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[12] TOTAL

[2] **1.** Consider the equation $A\mathbf{x} = \mathbf{b}$, where $A = \begin{bmatrix} 1 & 3 & 4 \\ 3 & 2 & 5 \\ 2 & 0 & 5 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix}$.

CRAMER'S RULE

If $\det(A) = -21$, what is the value of x_1?

$$x_1 = \frac{1}{-21} \det\begin{bmatrix} -2 & 3 & 4 \\ 0 & 2 & 5 \\ 0 & 0 & 5 \end{bmatrix} = \frac{-1}{21}(-2)(2)(5) = \frac{20}{21}.$$

[2] **2.** Let $A = \begin{bmatrix} a & b & c \\ 2 & d & -3 \\ -4 & e & f \end{bmatrix}$. Suppose that $\det(A) = 4$.

ADJOINT FORMULA

What is the value of f so that the entry in row 2, column 1, of A^{-1} is $\frac{1}{2}$?

$$\frac{1}{2} = (A^{-1})_{21} = \frac{1}{\det A}(-1)^{1+2} M_{12} = \frac{-1}{4}\det\begin{pmatrix} 2 & -3 \\ -4 & f \end{pmatrix} = -\frac{1}{4}(2f - 12)$$

$$-2 = 2f - 12, \quad 10 = 2f, \quad f = 5.$$

[8] **3.** Consider the vectors $\mathbf{u} = (0, 3, -1, -1)$, $\mathbf{v} = (-3, 2, -1, 4)$, and $\mathbf{w} = (2, 2, -1, -1)$.
Find:

(a) $\|\mathbf{u} - \mathbf{w}\|$.

$$\|\underline{u} - \underline{w}\| = \|(0,3,-1,-1) - (2,2,-1,-1)\| = \|(-2,1,0,0)\|$$
$$= \sqrt{2^2 + 1^2 + 0 + 0} = \sqrt{5}.$$

(b) A unit vector in the direction of $\mathbf{u} - \mathbf{w}$. divide by the norm of $\underline{u} - \underline{w}$

$$\frac{1}{\sqrt{5}}(-2, 1, 0, 0) = \left(\frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0, 0\right)$$

(c) $d(\mathbf{v}, \mathbf{w})$. $= \|\mathbf{v} - \mathbf{w}\| = \|(-5, 0, 0, 5)\|$

$$= \sqrt{25 + 25} = 5\sqrt{2}.$$

(d) $\mathbf{u} \cdot \mathbf{v}$. $=$

$$= (0, 3, -1, -1) \cdot (-3, 2, -1, 4)$$
$$= 0 + 6 + 1 - 4 = 3.$$