MATH 1300(A02) QUIZ 3 March 04, 2016 B07

NAME SOLUTIONS ID Number

INSTRUCTIONS: Answer the following questions in the spaces provided below.

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[12] TOTAL

[2] **1.** Consider the equation $A\mathbf{x} = \mathbf{b}$, where $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 4 & 3 \\ 3 & 5 & 4 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$. (CRAMER'S RULE)

If $\det(A) = -3$, what is the value of x_3?

$$x_3 = \frac{1}{-3}\det\begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 3 & 5 & 2 \end{bmatrix} = -\frac{1}{3}(1)(4)(2) = -\frac{8}{3}$$

[2] **2.** Let $A = \begin{bmatrix} a & 3 & 2 \\ b & c & d \\ e & 4 & 3 \end{bmatrix}$. Suppose that $\det(A) = -5$. (ADJOINT FORMULA)

What is the entry in row 1, column 2, of A^{-1}?

$$(A^{-1})_{12} = \frac{1}{\det A}(-1)^{2+1}M_{21} = \frac{1}{-5}(-1)\det\begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix} = \frac{1}{5}(9-8) = \frac{1}{5}.$$

[8] **3.** Consider the vectors $\mathbf{u} = (1, 0, -2, 2)$, $\mathbf{v} = (2, 2, 1, 4)$, and $\mathbf{w} = (2, -2, 1, 0)$.

Find:

(a) $\|\mathbf{w}\|$.

$$\|w\| = \sqrt{2^2 + (-2)^2 + 1^2 + 0^2} = \sqrt{9} = 3$$

(b) A unit vector in the direction of $\mathbf{w}$. divide by the norm of w.

$$\frac{1}{3}(2, -2, 1, 0) = \left(\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}, 0\right)$$

(c) $d(\mathbf{u}, \mathbf{v} - \mathbf{w})$. $= \|u - (v - w)\|$

$$= \|(1, 0, -2, -2)\| = \sqrt{1 + 0 + 4 + 4}$$

$$= \sqrt{9} = 3.$$

(d) $\mathbf{w} \cdot \mathbf{v}$.

$$= (2, -2, 1, 0) \cdot (2, 2, 1, 4)$$

$$= 4 - 4 + 1 + 0 = 1.$$