

NAME SOLUTIONS ID Number _____

INSTRUCTIONS: Answer the following questions in the spaces provided below.

[12] TOTAL

- [2] 1. Consider the equation $Ax = b$, where $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & 0 \\ 3 & 4 & 5 \end{bmatrix}$ and $b = \begin{bmatrix} 0 \\ -2 \\ 3 \end{bmatrix}$. CRAMER'S RULE

If $\det(A) = -5$, what is the value of x_2 ?

$$x_2 = \frac{1}{-5} \det \begin{bmatrix} 1 & 0 & 0 \\ 2 & -2 & 0 \\ 3 & 3 & 5 \end{bmatrix} = \frac{1}{-5} (1)(-2)(5) = 2.$$

- [2] 2. Let $A = \begin{bmatrix} 2 & b & -3 \\ 3 & c & 3 \\ d & 2 & e \end{bmatrix}$. Suppose that $\det(A) = -4$.

ADJOINT FORMULA

What is the entry in row 2, column 3, of A^{-1} ?

$$(A^{-1})_{23} = \frac{1}{\det A} (-1)^{3+2} M_{32} = \frac{1}{(-4)} (-1) \det \begin{bmatrix} 2 & -3 \\ 3 & 3 \end{bmatrix} = \frac{1}{4} (6+9) = \frac{15}{4}$$

- [8] 3. Consider the vectors $u = (3, -1, 2, -1)$, $v = (2, -1, 0, 2)$, and $w = (-3, -2, -1, 1)$. Find:

(a) $\|v\|$.

$$\|v\| = \sqrt{2^2 + (-1)^2 + 0^2 + 2^2} = \sqrt{9} = 3.$$

(b) A unit vector in the direction of v .

Divide by the norm of v .

$$\frac{1}{3} (2, -1, 0, 2) = \left(\frac{2}{3}, -\frac{1}{3}, 0, \frac{2}{3} \right)$$

(c) $d(u+w, v)$.

$$= \|(u+w) - v\| = \|(8, 0, 3, 0)\| = \sqrt{64+9} = \sqrt{73}$$

(d) $u \cdot v$.

$$= (3, -1, 2, -1) \cdot (2, -1, 0, 2)$$

$$= 6 + 1 + 0 - 2 = 5$$