

NAME SOLUTIONS ID Number _____

INSTRUCTIONS: Answer the following questions in the spaces provided below.

[12] TOTAL

- [2] 1. Consider the equation $A\mathbf{x} = \mathbf{b}$, where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 0 & 5 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$ CRAMER'S RULE

If $\det(A) = -8$, what is the value of x_1 ?

$$x_1 = \frac{1}{-8} \det \begin{bmatrix} 2 & 2 & 3 \\ 0 & 3 & 4 \\ 0 & 0 & 5 \end{bmatrix} = -\frac{1}{8}(2 \cdot 3 \cdot 5) = -\frac{15}{4}.$$

- [2] 2. Let $A = \begin{bmatrix} 2 & 1 & e \\ d & e & 0 \\ 3 & 3 & f \end{bmatrix}$. Suppose that $\det(A) = 5$.

What is the entry in row 3, column 2, of A^{-1} ?

ADJOINT FORMULA

$$(A^{-1})_{32} = \frac{1}{\det A} (-1)^{3+2} M_{23} = \frac{1}{5}(-1) \det \begin{bmatrix} 2 & 1 \\ 3 & 3 \end{bmatrix} = -\frac{1}{5}(6-3) = -\frac{3}{5}.$$

- [8] 3. Consider the vectors $\mathbf{u} = (1, 0, -2, 2)$, $\mathbf{v} = (-1, 2, 2, 4)$, and $\mathbf{w} = (2, 2, -1, -1)$.
Find:

(a) $\|\mathbf{u}\|$.

$$\|\mathbf{u}\| = \sqrt{1^2 + 0^2 + (-2)^2 + 2^2} = \sqrt{9} = 3$$

(b) A unit vector in the direction of $\mathbf{u}$. (divide by the norm of $\mathbf{u}$)

$$\frac{1}{3}(1, 0, -2, 2) = \left(\frac{1}{3}, 0, -\frac{2}{3}, \frac{2}{3}\right)$$

(c) $d(\mathbf{v}, \mathbf{w})$. $\Rightarrow \|\mathbf{v} - \mathbf{w}\| = \|(-3, 0, 3, 5)\|$
 $= \sqrt{9 + 0 + 9 + 25} = \sqrt{43}.$

(d) $(\mathbf{u} + \mathbf{v}) \cdot \mathbf{w}$.

$$= (0, 2, 0, 6) \cdot (2, 2, -1, -1) =$$

$$= 0 + 4 + 0 - 6 = -2.$$