MATH 1300(A02) QUIZ 2 February 08, 2016 B08

NAME Solutions ID Number

INSTRUCTIONS: Answer the following questions in the spaces provided below.

[12] TOTAL

[3] **1.** Let A be a square matrix. List any three statements (from Sections 1.4 to 1.7) equivalent to "A is invertible (non-singular)".

(b) $A\underline{x} = \underline{0}$ has only the trivial solution

(c) The reduced row echelon form of A is I_n

(d). A is expressible as a product of elementary matrices

(e). $A\underline{x} = \underline{b}$ is consistent for every $n\times 1$ matrix $\underline{b}$

(f) $A\underline{x} = \underline{b}$ has exactly one solution for every $n\times 1$ matrix $\underline{b}$

* you don't have to mention n in (c), (e) (f)

[4] **2.** Determine all vectors $\mathbf{b} = (b_1, b_2, b_3)$ such that the following system is consistent:

$$\begin{aligned} x - 2y + z &= b_1 \\ 2x - 3y + 4z &= b_2 \\ 3x - 4y + 7z &= b_3 \end{aligned}$$

Best solution is by row reduction. Full credit for correct solution by any method.

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & b_1 \\ 2 & -3 & 4 & b_2 \\ 3 & -4 & 7 & b_3 \end{array}\right] \begin{array}{l} \\ R_2 - 2R_1 \\ R_3 - 3R_1 \end{array} \quad \left[\begin{array}{ccc|c} 1 & -2 & 1 & b_1 \\ 0 & 1 & 2 & b_2 - 2b_1 \\ 0 & 2 & 4 & b_3 - 3b_1 \end{array}\right] R_3 - 2R_2$$

Only the 3rd row matters for consistency:

$[0 \quad 0 \quad 0 \mid (b_3 - 3b_1) - 2(b_2 - 2b_1)]$.

The constant must be 0, so $b_3 - 3b_1 - 2b_2 + 4b_1 = 0$

So $b_3 = 2b_2 - b_1$.

$(b_1, b_2, b_3) = (b_1, b_2, 2b_2 - b_1)$

[5] **3.** Find A^{-1}, where $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 2 & 2 \\ 3 & 3 & -5 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 & 1 & 0 \\ 3 & 3 & -5 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \tfrac{1}{2}R_2 \\ R_3 - 3R_1 \end{array} \left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1/2 & 0 \\ 0 & -3 & -2 & -3 & 0 & 1 \end{array}\right] \begin{array}{l} R_1 - 2R_2 \\ \\ R_3 + 3R_2 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -3 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & -3 & 3/2 & 1 \end{array}\right] \begin{array}{l} R_1 + 3R_3 \\ R_2 - R_3 \\ \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -8 & 7/2 & 3 \\ 0 & 1 & 0 & 3 & -1 & -1 \\ 0 & 0 & 1 & -3 & 3/2 & 1 \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} -8 & 7/2 & 3 \\ 3 & -1 & -1 \\ -3 & 3/2 & 1 \end{bmatrix}.$$