MATH 1300(A02) QUIZ 2 February 05, 2016 **B07**

NAME SOLUTIONS ID Number

INSTRUCTIONS: Answer the following questions in the spaces provided below.

[12] TOTAL

[3] **1.** Show that for any square matrix A, if A is invertible then so is A^T.

We show that $(A^T)^{-1} = (A^{-1})^T$.

For $A^T \cdot (A^{-1})^T = (A^{-1} \cdot A)^T = I^T = I$.

Therefore $(A^{-1})^T = (A^T)^{-1}$.

[4] **2.** Determine all vectors $\mathbf{b} = (b_1, b_2, b_3)$ such that the following system is consistent:

$$\begin{aligned} x + 2y - z &= b_1 \\ 2x + 3y + 4z &= b_2 \\ 3x + 7y - 2z &= b_3 \end{aligned}$$

Best solution is by row reduction. Full credit for correct solution by any method.

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & b_1 \\ 2 & 3 & 4 & b_2 \\ 3 & 7 & -2 & b_3 \end{array}\right] \begin{array}{l} \\ R_2 - 2R_1 \\ R_3 - 3R_1 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 2 & -1 & b_1 \\ 0 & -1 & 6 & b_2 - 2b_1 \\ 0 & 1 & 1 & b_3 - 3b_1 \end{array}\right] R_3 + R_2$$

Only the third row matters for consistency

$[0 \quad 0 \quad 7 \mid (b_2 - 2b_1) + (b_3 - 3b_1)]$.

Since there is a non-zero entry in the coefficient part, this system is always consistent

[5] **3.** Find A^{-1}, where $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & 6 \\ 2 & 2 & -5 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 3 & 6 & 0 & 1 & 0 \\ 2 & 2 & -5 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ 1/3 R_2 \\ R_3 - 2R_1 \end{array} \quad \left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1/3 & 0 \\ 0 & -2 & -3 & -2 & 0 & 1 \end{array}\right] \begin{array}{l} R_1 - 2R_2 \\ \\ R_3 + 2R_2 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -5 & 1 & -2/3 & 0 \\ 0 & 1 & 2 & 0 & 1/3 & 0 \\ 0 & 0 & 1 & -2 & 2/3 & 1 \end{array}\right] \begin{array}{l} R_1 + 5R_3 \\ R_2 - 2R_3 \\ \\ \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -9 & 8/3 & 5 \\ 0 & 1 & 0 & 4 & -1 & -2 \\ 0 & 0 & 1 & -2 & 2/3 & 1 \end{array}\right]$$

Therefore $A^{-1} = \begin{bmatrix} -9 & 8/3 & 5 \\ 4 & -1 & -2 \\ -2 & 2/3 & 1 \end{bmatrix}$