MATH 1300(A02) QUIZ 2 February 05, 2016 B06

NAME SOLUTIONS. ID Number__________

INSTRUCTIONS: Answer the following questions in the spaces provided below.

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[12] TOTAL

[3] **1.** Show that for any shape of matrix A, A^TA is defined and symmetric.

Let A be $m \times n$. Then A^T is $n \times m$.

So we have $\underbrace{A^T \quad A}_{(n\times m)\ (m\times n)}$ and multiplication is defined

Then $(A^TA)^T = A^T A^{TT} = A^TA$,

So $A^T\cancel{A}$ is symmetric.

[4] **2.** Determine all vectors $\mathbf{b} = (b_1, b_2, b_3)$ such that the following system is consistent:

$$\begin{aligned} x - y + 2z &= b_1 \\ 2x - y + 5z &= b_2 \\ 3x - 2y + 7z &= b_3 \end{aligned}$$

Best solution is by row reduction. Full credit for correct solution by any method

$$\left[\begin{array}{ccc|c} 1 & -2 & 2 & b_1 \\ 2 & -1 & 5 & b_2 \\ 3 & -2 & 7 & b_3 \end{array}\right] \begin{array}{l} \\ R_2 - 2R_1 \\ R_3 - 3R_1 \end{array} \quad \left[\begin{array}{ccc|c} 1 & -2 & 2 & b_1 \\ 0 & 1 & 1 & b_2 - 2b_1 \\ 0 & 1 & 1 & b_3 - 3b_1 \end{array}\right] \begin{array}{l} \\ \\ R_3 - R_2 \end{array}$$

For consistency, only the 3rd row matters:

$$\left[\begin{array}{ccc|c} 0 & 0 & 0 & (b_3 - 3b_1) - (b_2 - 2b_1) \end{array}\right]$$

So $b_3 - 3b_1 - b_2 + 2b_1 = 0$, or $b_3 = b_2 - b_1$.

So $(b_1, b_2, b_3) = (b_1, b_2, b_1 - b_2)$ for any b_1, b_2

[5] **3.** Find A^{-1}, where $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 2 & -2 \\ 3 & 3 & 1 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 2 & -2 & 0 & 1 & 0 \\ 3 & 3 & 1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \tfrac{1}{2}R_2 \\ R_3 - 3R_1 \end{array} \quad \left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1/2 & 0 \\ 0 & -3 & 4 & -3 & 0 & 1 \end{array}\right] \begin{array}{l} R_1 - 2R_2 \\ \\ R_3 + 3R_2 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & -3 & 3/2 & 1 \end{array}\right] \begin{array}{l} R_1 - R_3 \\ R_2 + R_3 \\ \end{array} \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 4 & -5/2 & -1 \\ 0 & 1 & 0 & -3 & 2 & 1 \\ 0 & 0 & 1 & -3 & 3/2 & 1 \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} 4 & -5/2 & -1 \\ -3 & 2 & 1 \\ -3 & 3/2 & 1 \end{bmatrix}$$