MATH 1300(A02) QUIZ 2 February 05, 2016 B05

NAME SOLUTIONS ID Number

INSTRUCTIONS: Answer the following questions in the spaces provided below.

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[12] TOTAL

[3] **1.** Let A be a square matrix. List any three statements (from Sections 1.4 to 1.7) equivalent to "A is invertible (non-singular)".

(b) $A\underline{x} = \underline{0}$ has only the trivial solution.
(c) The reduced row echelon form of A is I_n.
(d) A is expressible as a product of elementary matrices
(e) $A\underline{x} = \underline{b}$ is consistent for every $n \times 1$ matrix $\underline{b}$.
(f) $A\underline{x} = \underline{b}$ has exactly one solution for every $n \times 1$ matrix $\underline{b}$

* you don't have to mention n in (c), (e), (f).

[4] **2.** Determine all vectors $\mathbf{b} = (b_1, b_2, b_3)$ such that the following system is consistent:

$$\begin{aligned} x + y + z &= b_1 \\ 2x + 3y + 4z &= b_2 \\ 3x + 4y + 5z &= b_3 \end{aligned}$$

Best solution is by row reduction. Full credit for correct solution by any method.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & b_1 \\ 2 & 3 & 4 & b_2 \\ 3 & 4 & 5 & b_3 \end{array}\right] \begin{array}{l} R_2 - 2R_1 \\ R_3 - 3R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & b_1 \\ 0 & 1 & 2 & b_2 - 2b_1 \\ 0 & 1 & 2 & b_3 - 3b_1 \end{array}\right] R_3 - R_2.$$

Only the 3rd row matters: $[0 \;\; 0 \;\; 0 \mid (b_3 - 3b_1) - (b_2 - 2b_1)]$

For consistency the constant must be 0:

$b_3 - 3b_1 + b_2 + 2b_1 = 0$, or $b_3 = b_1 - b_2$

So $(b_1, b_2, b_3) = (b_1, b_2, b_1 - b_2)$, for any values of b_1, b_2

[5] **3.** Find A^{-1}, where $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & 3 \\ 2 & 2 & -3 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 3 & 3 & 0 & 1 & 0 \\ 2 & 2 & -3 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \tfrac{1}{3}R_2. \\ R_3 - 2R_2 \end{array} \left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1/3 & 0 \\ 0 & -2 & -1 & -2 & 0 & 1 \end{array}\right] \begin{array}{l} R_1 - 2R_2 \\ \\ R_3 + 2R_2 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -3 & 1 & -2/3 & 0 \\ 0 & 1 & 1 & 0 & 1/3 & 0 \\ 0 & 0 & 1 & -2 & 2/3 & 1 \end{array}\right] \begin{array}{l} R_1 + 3R_3 \\ R_2 - R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -5 & 4/3 & 3 \\ 0 & 1 & 0 & 2 & -1/3 & -1 \\ 0 & 0 & 1 & -2 & 2/3 & 1 \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} -5 & 4/3 & 3 \\ 2 & -1/3 & -1 \\ -2 & 2/3 & 1 \end{bmatrix}$$