

NAME SOLUTIONS ID Number _____

INSTRUCTIONS: Answer the following questions in the spaces provided below.

[12] TOTAL

- [2] 1. Write the augmented matrix corresponding to the following system in the space to the right:

$$\begin{cases} x_1 + x_2 = 3x_3 - 4 \\ x_2 + x_3 = -2 + 4x_1 \\ x_3 + x_1 = -1 + 2x_2 \end{cases} \quad \left[\begin{array}{ccc|c} 1 & 1 & -3 & -4 \\ -4 & 1 & 1 & -2 \\ 1 & -2 & 1 & -1 \end{array} \right]$$

- [2] 2. Consider the following two augmented matrices. Below each, describe, in words or symbols, the *next* elementary row operation to be performed on each, according to the method of Gaussian elimination with back substitution, as described in the textbook and in class. (Do not compute anything).

(a) $\left[\begin{array}{ccc|c} 1 & 0 & 2 & -3 \\ 0 & 0 & 2 & 3 \\ 0 & 3 & 1 & 2 \end{array} \right]$

(b) $\left[\begin{array}{ccc|c} 1 & 0 & 2 & -1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & -2 & 1 \end{array} \right]$

$$R_2 \leftrightarrow R_3$$

$$R_3 \leftarrow R_3 + 2R_2$$

- [6] 3. The following matrix is the reduced row echelon form of a system of linear equations with variables x, y, z . Write the general solution of the system in the space to the right.

$$\left[\begin{array}{ccc|c} 1 & -3 & 0 & 2 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{array} \right] \quad \begin{array}{l} y \text{ is free:} \\ y = t \end{array} \quad \begin{cases} x = 2 + 3t \\ y = t \\ z = 5 \end{cases}$$

$$\text{or: } (x, y, z) = (2 + 3t, t, 5)$$

- [4] 4. Let $A = \begin{bmatrix} 2 & -1 & 3 \\ -3 & 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -2 \\ 2 & -3 \\ 4 & 2 \end{bmatrix}$. For each of the following expressions, calculate it if it is defined, and explain why it is not defined otherwise.

(a) $2A - 3B$

NOT DEFINED: $2A$ and $3B$ do not have the same shape.

(b) $B + 3A^T$

$$\begin{bmatrix} 9 & -11 \\ -1 & 3 \\ 13 & -1 \end{bmatrix}$$

- [2] 5. Let $A = \begin{bmatrix} 3 & -1 & 4 \\ -1 & 5 & -9 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 7 \\ 1 & 8 \\ 2 & -8 \end{bmatrix}$. Find row 1 of AB .

$$[1 \quad -19]$$