MATH 1300 (A02), January 12, 2016

Gaussian elimination

Start with Row 1 and Column 1.

1. Starting with the current column, **search** if necessary from left to right to find the first column that has a non-zero entry in the current row or below. Reset the column number.

2. If necessary, **interchange** the current row with a lower row in order to get a non-zero entry a in the current row/current column. This entry is called the *pivot*. If all the rows including the current row or below are zero, you are done.

3. Multiply the current row by $1/a$ to make the pivot equal to 1.

4. For each row *below* the current row which has some entry $c \neq 0$ in the current column, add $(-c)$ times the current row to that row, in order to produce a zero entry in the current column.

5. Add one to the current column number and one to the current row number. If you "fall out" of the matrix (either at the bottom or at the right) you are done. Otherwise, return to step 1 and continue.

Done: The result is an augmented matrix in row-echelon form.

Back substitution

Start with the final pivot found, set the current row and column to the location of that pivot.

1. For each row *above* the current row which has some entry $c \neq 0$ in the current column, add $(-c)$ times the current row to that row, in order to produce a zero entry in the current column.

2. Move to the next pivot to the left, resetting the row number and column number. If you "fall out" of the matrix, you are done. Otherwise, return to step 1.

Done: The result is an augmented matrix in reduced row-echelon form.

Interpretation

1. If the row-echelon form has a pivot in the column of constants (that is, a row entirely of zeroes except for the constant entry) then the system was *inconsistent*: it has no solutions

2. If every column (corresponding to a variable) has a pivot entry, then the system is consistent and has a *unique solution*, which can be read off directly from the reduced row-echelon form.

3. Otherwise, the system is *consistent with infinitely many solutions.* A *parametric* description of the solution can be read off from the reduced row-echelon form.

Parametric solution

(Case 3 above). Variables corresponding to "leading ones" (pivots) are called *leading variables.* Other variables are called *free variables.*

1. Choose a parameter name for each free variable and set each free variable equal to its corresponding parameter.

2. Use the pivot row equations to solve for each leading variable in terms of the parameter variables.

This can be done with the row-echelon matrix, but is easier with the reduced row-echelon matrix.