

Rings and Modules Seminar, 14 March 2017.

Single Palindromic Axioms for Groups

Inspired by a recent article [1] on palindromes, I wanted to find a single palindromic axiom for, say, the class of all groups treated as algebras of type (2,1) with the binary operation of group multiplication $*$ and the unary group inverse \bar{x} . Here are a few such axioms characterizing various well-known varieties of groups.

1. Boolean groups.

$$((x * y) * z) * (y * z) = x \wedge x = (z * y) * (z * (y * x))$$

2. Abelian groups.

$$(y * (x * z)) * \overline{(z * y)} = x \wedge x = \overline{(y * z)} * ((z * x) * y)$$

3. The class of all groups

$$((x * y) * z) * \overline{(y * z)} = x \wedge x = \overline{(z * y)} * (z * (y * x))$$

4. Symmetric difference in abelian groups

$$(x * (y * z)) * ((u * y) * (u * z)) = x \wedge x = ((z * u) * (y * u)) * ((z * y) * x)$$

References

- [1] Marco Buratti, Happy 2017, *Bulletin ICA*, January 2017.
- [2] G. Grätzer and R. Padmanabhan, Symmetric difference in abelian groups, *Pacific J. Math.* 74 (1978) 339-347
- [3] N.S. Mendelsohn and R. Padmanabhan, A single identity for Boolean groups and Boolean rings. *J. Algebra* 20 1972 78–82.
- [4] R. Padmanabhan and W. McCune, *Tarski Theorems*, Argonne National Labs, University of Chicago, under Contract No. W-31-109-ENG-38, 2004.