

The Union-closed Sets Conjecture via Finite Lattices II: Dismantling Finite Lattices

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ABSTRACT. Please refer to the notes handed out at my first talk. In this second lecture, we start with $\mathcal{L}(n)$ being the class of all finite lattices with exactly n join irreducible elements (such lattices have at most 2^n elements), with $\mathcal{JFR}(n)$ being the set of all \cap -closed subsets of $\mathfrak{P}(n)$ containing \emptyset and $[n]$ (all of which also belong to $\mathcal{L}(n)$), and with the Subscript Embedding Theorem which asserts that every member of $\mathcal{L}(n)$ is order isomorphic (and so, lattice isomorphic) to a member of $\mathcal{JFR}(n)$. In this lecture, I will show how to obtain every member of $\mathcal{JFR}(n)$ (each of which is a subset of $\mathfrak{P}(n)$) by starting with $\mathfrak{P}(n)$ and serially removing a \cap irreducible element.