The Union-closed Sets Conjecture via Finite Lattices I

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ABSTRACT. The Union-closed Sets Conjecture asserts that for any non-empty but possibly infinite set S and for any finite set U of subsets of S that is closed under the union of sets in U, there is some $i \in S$ such that the number of sets in U containing i is at least half of $|U|$. This is known to be equivalent to the assertion that in any finite lattice $\boldsymbol{L}$ there is a join irreducible element j such that the size of the principal filter generated by j is at most half the size of $\boldsymbol{L}$. In this first talk, I will explain the algebra behind this equivalence and suggest a structure theory for finite lattices towards proving the Conjecture.