

Locally Projective Modules,  
Zimmermann-Huisgen's "Property A"  
and the Elementary Dual

(Extended abstract)

T. G. Kucera, University of Manitoba, Winnipeg, Manitoba  
Ph. Rothmaler, Bronx Community College, City University of New York

In this series of talks I will work through the algebraic details of an interesting but very stubborn example in the (infinitary) model theory of modules that has been bedeviling the two of us for some time now.

B. Zimmermann-Huisgen's "Property A" [2] is a quite technical but computationally interesting formulation equivalent to a property of modules called "local projectivity".

In fact, "Property A" leads to a natural axiomatization of local projectivity by implications of infinitary pp formulss; and in nice enough a form that "Property A" is easily seen to have a (generalized) elementary dual in the sense of my ongoing joint work with Rothmaler [1]. It is possible that "Dual A" is a novel algebraic property of modules.

The natural algebraic meaning of the property "Dual A" is obscure to say the least. It is the goal of this series of lectures is to give a detailed enough account of the algebraic manipulations involved in describing "Dual A" that perhaps with the aid of the audience I will finally be able to make algebraic sense of it.

Topics, not necessarily in order:

1. Homomorphisms and systems of equations; projectivity and injectivity, constructing a homomorphism
2. Property 'A', local projectivity, and their equivalence.
3. Axiomatization of Property 'A'.
4. Basic constructions of elementary duality
5. Simplifying 'Dual A'
6. But what does it mean?

## References

- [1] T. G. Kucera and Ph. Rothmaler. Infinitary properties of modules. unpublished private research notes, multiple versions, August 2016 et seq.
- [2] Birge Zimmermann-Huisgen. Pure submodules of direct products of free modules. *Math. Ann.*, 224(3):233–245, 1976.