

Title

Tarski spectrum for group-like varieties.

Abstract

For a given equational theory Σ , Alfred Tarski defined the set $\nabla(\Sigma)$ as

$$\nabla(\Sigma) = \{|\Sigma_0|: \Sigma_0 \text{ is an independent base for } \Sigma\}$$

where $|\Sigma_0|$ is the cardinality of the set Σ_0 . Tarski's interpolation theorem states that $\nabla(\Sigma)$ is always an interval and that it is unbounded if $\Sigma \vdash f=x$ where the term f contains the variable x at least twice. Thus, in particular, for any equational theory Σ of groups, we have $\nabla(\Sigma) = [1, \omega)$. Tarski's 1968 proof is existential - he uses topological tools like closure operations to show the existence of an independent basis of cardinality n for all n . In this talk, we give constructive proofs for the Tarski's "unbounded theorem" for several examples of group-like varieties by constructing independent bases with n identities for all n .

References

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