THERE ARE JUST TWO POSITIVE UNIVERSAL CLASSES OF TOTALLY ORDERED ABELIAN GROUPS

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ABSTRACT. A *lattice ordered abelian group* (or abelian ℓ-group) is an algebra $\mathbf{G} = (G; \wedge, \vee, +, -, 0)$ such that:

(a) $(G; \wedge, \vee)$ is a (necessarily distributive) lattice with ordering denoted $\leq$,

(b) $(G; +, -, 0)$ is an abelian group,

(c) $(x \wedge y) + z \leq y + z$ (that is, addition is *order preserving*).

$\mathbf{G}$ is a *t-group* if it satisfies the positive universal sentence:

$$(x \leq y) \text{ OR } (y \leq x).$$

It is a classical result that every lattice ordered abelian group is a subdirect product of abelian t-groups.

The two positive universal classes of abelian t-groups are the trivial class of 1-element t-groups, axiomatized by $x = y$, and the class of all abelian t-groups. I will present a proof of this.