PETER LOLY, Physics and Astronomy

Eigenvalues of an Algebraic Family of Compound Magic Squares of Order $n = 3^l, l = 2, 3, \ldots$, and Construction and Enumeration of their Fundamental Numerical Forms. (Joint work with Ian Cameron.)

Compound magic squares (CMSs) of order mn, whose tiled subsquares of orders m and n are also magic squares (MSs having constant row, column and diagonal linesums within each subsquare), are found back to the 10th century for the case $m = n = 3$. Interesting results follow if they are considered as matrices.

Frierson gave a simple algebraic form for compounding from the unique pattern of third order to a general $n = 9$ CMS in The Monist in 1907, from which he showed 6 fundamental numerical forms using the complete set of integers 1...81. We extend Frierson's work, finding an algebraic description of a family of associative (antipodal sum pairs $n^2 + 1$) compound magic squares of orders $n = 3^l, l = 1, 2, \ldots$. In doing so we have firmly established two results previously stated by Bellew (1997), 90 fundamental numerical forms for $n = 27$, as well as its generalization for all l.

The present algebra then leads to a general formula for the eigenvalues of this family, which consists of the linesum eigenvalue and l signed pairs, for rank $2l + 1$.

For $n = 9$ the 8 possible orientations of each of the 9 tiled third order subsquares give rise to 6×8^9 distinct CMSs, most with increased rank. We resolve disparate factors of 8 of Trigg (1980) and Bellew for $n = 27$ with a new result by taking account of all orders of tiled subsquares, before generalizing this for all l.

In addition to the main part of the talk, which was presented at CMS2009, I will show some results from 'pouring water' over Lego models of magic squares and Sudokus, as well as some further algebraic gems from an archive of Frierson's work.